THE

KHAṆḌAKHĀDYAKA

AN ASTRONOMICAL TREATISE OF

BRAHMAGUPTA

TRANSLATED INTO ENGLISH
WITH AN INTRODUCTION, NOTES, ILLUSTRATIONS AND APPENDICES

BY

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TO

THE MEMORY OF

SIR ASUTOSH MOOKERJEE

THE FOUNDER OF RESEARCH STUDIES

IN THE

CALCUTTA UNIVERSITY

AS A TOKEN OF

SINCERE ESTEEM AND REGARD

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INTRODUCTION

"Brahmagupta holds a remarkable place in the history of Eastern civilisation. It was he who taught the Arabs astronomy before they became acquainted with Ptolemy; for the famous *Sindhind* of Arabian literature, frequently mentioned but not yet brought to light, is a translation of his *Brahmasiddhānta*; and the only other book on Indian astronomy, called *Alarkand*, which they knew, was a translation of his *Khaṇḍakhādyaka*."

Dr. E. C. Sachau, *Alberūnī's India*, Vol. II, p. 304.

In a paper "Āryabhaṭa the Father of Indian Epicyclic Astronomy," * it has been established that the scientific Hindu Astronomy was created by Āryabhaṭa I (476 A.D.). He was the teacher of two distinct systems of astronomy, one of which is called the *audayika* system, and the other the *ārdharātrika* system. In the first the astronomical day is taken to begin at sunrise at Laṅkā and in the other the same begins at the midnight of the same place. In the *Khaṇḍakhādyaka* Brahmagupta gives compendious rules for the calculation of longitudes, etc., of 'planets,' according to the *ārdharāṭrika* system of Āryabhata I.† It was this system that was used by Varāhamihira when he gave the epicyclic cast to the *Sūryasiddhānta* in his *Pañcasiddhāntikā*. For this the reader is referred to the present translator's papers "Āryabhaṭa's Lost Work"‡ and "Āryabhaṭa the Father of Indian Epicyclic Astronomy" already mentioned.

The question why Brahmagupta who was so bitter an opponent of Āryabhaṭa in his younger days (628 A.D.) climbed down to describe and teach one of the systems of Āryabhaṭa's astronomy in his sixty-seventh year (665 A.D.), cannot yet be properly answered. So great was Āryabhaṭa's fame that in spite of Brahmagupta's severe criticisms of the former in Chapter XI of his *Brāhma-sphuṭa-siddhānta*, it perhaps was undiminished and it was Āryabhaṭa who continued to be universally followed. Perhaps to meet this popular demand Brahmagupta in the *Khaṇḍakhādyka* took upon himself the task of simplifying Āryabhaṭa's *ārdharātrika* system and from the present translation it will appear that in this work he was

* Journal of the Department of Letters, Calcutta University, Vol. XVIII.
† *Khaṇḍakhādyaka*, Translation, I stanzas 1, 2 and 7.
‡ Bulletin, Calcutta Mathematical Society, Vol. XXII, Nos. 2 and 3.

eminently successful. But Brahmagupta could not be a mere simplifier or expounder.

The work *Khaṇḍakhādyaka* had two distinct parts, *viz.*, the *Khaṇḍakhādyaka* proper and the *Uttara Khaṇḍakhādyaka*. In the first part the astronomical constants are the same as those of Āryabhaṭa's *ārdharātrika* system, but the methods of spherical astronomy, calculations of eclipses and other topics are almost the same as in the *Brāhmasphuṭa-siddhānta*. The correction for parallax in calculating a solar eclipse is here an important illustration.* In the *Uttara Khaṇḍakhādyaka*, Brahmagupta gives corrections to the *Khaṇḍakhādyaka* proper. In it are to be found the neat and original methods of interpolation and correction to the longitudes of the aphelia, as also to the dimensions to the epicycles of apsis of the sun and the moon,† while a few additional chapters supply what else is necessary to the seven chapters of the first part, to make the whole a complete treatise on Hindu scientific astronomy. Later on will be detailed the additional matters treated of in this *Uttara* portion. It was perhaps through the influence of this supplementary part of the *Khaṇḍakhādyaka*, that Brahmagupta's great work, the *Brāhmasphuṭa-Siddhānta*, came to be valued among a distinct school of Indian astronomers; even now this *Siddhānta* of Brahmagupta forms the basis for the calculation of almanacs by astronomers of the orthodox school in Rajputana, Bombay and other places.

Was Āryabhaṭa the author of two distinct systems of astronomy?

This question has already been answered in the affirmative but I trust, though I have treated of this question in my papers already mentioned, it would perhaps be not out of place to restate in detail the reasons for this hypothesis. In his *Brāhmasphuṭa-siddhānta* Brahmagupta thus speaks of the two works of Āryabhaṭa:—

युगरविभगणाः ख्युघ्निति यत् प्रोक्तं तत्तयोर्युगं स्पष्टम्।
त्रिशतीरव्युदयानां तदन्तरं हेतुना केन॥

Brāhmasphuṭasiddhānta, XI, 5.

"As in both the works the number of the sun's revolutions is spoken of as 4,320,000 years, their planatary cycle is clear, *i.e.*, of 4,320,000 years. Why then is there a difference of 300 civil days in the same cycle of the two books?"

* Translation, Chapter V, *Brāhma-sphuṭa-siddhānta*, XI, 23-25.

† Translation, Chapter IX.

Again in stanza 13 of the same chapter he says,

अधिकैः शतैश्चतुर्भिर्वर्षसहस्रैश्चतुर्दशभिरेकः।
युगयुगतैर्दिनवारान्तरमौदयिकार्द्धरात्रिकयोः॥

"In 14,400 years elapsed of the *Mahāyuga*, there is produced a difference of one day in the *audayika* and *ārdharātrika* systems."

Varāhamihira in his *Pañcasiddhāntikā*, XV, 20, writes—

लङ्कार्द्धरात्रसमये दिनप्रवृत्तिं जगाद चार्य्यभटः।
भूयः स एव सूर्य्योदयात् प्रभृत्याह लङ्कायाम्॥

Thibaut translates it thus:—

"Āryabhaṭa maintains that the beginning of the day is to be reckoned from midnight at Laṅkā; and the same teacher again says that the day begins from sunrise at Laṅkā."

Thus from the writings of Brahmagupta and Varāhamihira, it is clear that Āryabhaṭa I was the author of both the *audayika* and *ārdharātrika* systems of astronomy. In Varāhamihira's stanza the phrase स एव (=he undoubtedly) is of special significance. It removes the least doubt as to Āryabhaṭa's authorship of both these systems. These *audayika* and *ārdharātrika* astronomical constants are respectively to be found from the *Āryabhaṭīya* and may be deduced from the *Khaṇḍakhādyaka*. The following is the comparative view of the constants of the two systems along with those of the *Sūrya-siddhānta* of Varāhamihira and of the modern *Sūryyasiddhānta*.

(1) *Planetary Revolutions in a Mahāyuga of 4,320,000 Years.*

	According to *Āryabhaṭīya.*	According to *Khaṇḍa-khādyaka.*	According to *Sūryasiddhānta* of Varāha.	According to the Modern *Sūryasiddhānta.*
Of Moon	57753336	57753336	57753336	57753336
,, Sun	4320000	4320000	4320000	3320000
,, Mars	2296824	2296824	2296824	2296832
,, Jupiter	364224	364220	364220	364220
,, Saturn	146564	146564	146564	145568
,, Moon's apogee.	488219	488219	488219	488203
,, Venus	7022388	7022388	7022388	7022376
,, Mercury	17937020	17937000	17937000	17937060
,, Moon's nodes.	232226	232226	232226	232238

(2) *Longitudes of the Apogees of the Orbits of Planets.*

	According to *Āryabhaṭīya.*	According to *Khaṇḍa-khādyaka.*	According to *Sūryasiddhānta* of Varāha.	According to the Modern *Sūryasiddhānta.*
Of Sun	78°	80°	80°	77°17′
,, Mercury	210°	220°	220°	etc., have to be calculated from the data given in the text.
,, Venus	90°	80°	80°	
,, Mars	118°	110°	110°	
,, Jupiter	180°	160°	160°	
,, Saturn	236°	240°	240°	

(3) *Dimensions of the Epicycles of Apsis.*

	According to *Āryabhaṭīya.*	According to *Khaṇḍa-khādyaka.*	According to *Sūryasiddhānta* of Varāha.	According to the Modern *Sūryasiddhānta.*
Of Sun	13°30′	14°	14°	13⅔° to 14°
,, Moon	31°30′	31°	31°	31⅔° to 32°
,, Mercury	22½° to 31½°	28°	28°	28° to 30°
,, Venus	9° to 18°	14°	14°	11° to 12°
,, Mars	63° to 81°	70°	70°	72° to 75°
,, Jupiter	31½° to 36½°	32°	32°	32° to 33°
,, Saturn	40½° to 58½°	60°	60°	48° to 49°

(4) *Dimensions of the Śīghra Epicycles* (i.e., *of Conjunctions*).

	According to *Āryabhaṭīya.*	According to *Khaṇḍa-khādyaka.*	According to *Sūryasiddhānta* of Varāha.	According to the Modern *Sūryasiddhānta.*
Of Saturn	36½° to 40°	40°	40°	39° to 40°
,, Jupiter	67½° to 72°	72°	72°	70° to 72°
,, Mars	229½° to 239½°	234°	234°	232° to 235°
,, Venus	256½° to 265½°	260°	260°	260° to 262°
,, Mercury	130½° to 139½°	132°	132°	132° to 133°

(5) *Longitudes of the Nodes of the Orbits of Planets.*

	According to *Āryabhaṭīya.*	According to *Khaṇḍa-khādyaka.*	According to *Sūryasiddhānta* of Varāha.	According to the Modern *Sūryasiddhānta.*
Of Saturn	40°	40°	Not stated in the text.	Have to be calculated from the data of the text.
,, Jupiter	20°	20°		
,, Mars	80°	80°		
,, Venus	60°	60°		
,, Mercury	100°	100°		

(6) *Orbital Inclinations (Geocentric) to the Ecliptic.*

	According to *Āryabhaṭīya.*	According to *Khaṇḍa-khādyaka.*	According to *Sūryasiddhānta* of Varāha.	According to the Modern *Sūryasiddhānta.*
Of Mars	90′	90′	10′	90′
,, Mercury	120′	120′	135′	120′
,, Jupiter	60′	60′	101′	60′
,, Venus	120′	120′	101′	120′
,, Saturn	120′	120′	135′	100′
(7) Number of civil days in a *Mahāyuga* of 4,320,000 years.	1577917500	1577917800	1577917800	1577917828
(8) Beginning of the astronomical day.	Sunrise at Laṅkā.	Midnight at Laṅkā.	Midnight at Laṅkā.	Midnight at Laṅkā.

Dr. Bibhutibhushan Datta, late of the Calcutta University College of Science, has obtained copies of the Madras Government manuscripts of the *Mahābhāskarīya* and the *Laghubhāskarīya*, two astronomical works describing Āryabhaṭa's astronomy, composed evidently by one Bhāskara. The former of these books contains a passage which corroborates the fact that Āryabhaṭa I was the author of both the *audayika* and the *ārdharātrika* systems of astronomy. This Bhāskara whom we should designate as Bhāskara I, was probably a direct pupil of Āryabhaṭa I, as we learn from Pṛthudakasvāmī's statement—

"भास्करादीनामेवं भवतु तैर्न बुद्धस्तदभिप्रायः"

Pṛthūdaka's Comm. on *B. S. Siddhānta*, XI, 26.

"Such a mistake may have been made by Bhāskara and others; they have not understood his (Āryabhaṭa's) intention."

The passage in the *Mahābhāskarīya*, giving the constants of the *ārdharātrika* system runs as follows:—

निबन्धः कर्म्मणां प्रोक्तो योऽसावौदयिको विधिः ।
अर्द्धरात्रेस्त्वयं सर्व्वो यो विशेषः स कथ्यते ॥२१॥
त्रिशती भूदिने क्षेप्या ह्यवमेभ्यो विशोध्यते ।
ज्ञगुर्व्वोर्भगणेभ्योऽपि विंशतिश्च ततोऽब्धयः ॥२२॥*
अष्टिः शतगुणा व्यासो योजनानां भुवो रवेः ।
खाष्टाब्ध्यङ्गानि शीतांशोः शून्यवस्वब्धयस्तथा ॥२३॥
वह्निन्द्रियगुणच्छिद्रवस्वङ्गानि विभावसोः ।
अङ्गाङ्गेष्वेकभूतानि चन्द्रकर्णः प्रकीर्त्तितः ॥२४॥
अष्टिरष्ट जिना रुद्रा विंशतिर्द्व्यधिकाः क्रमात् ।
दशघ्ना गुरुशुक्रार्किभौमशाज्ञि स्वमन्दजाः ॥२५॥
मन्दवृत्तानि द्वात्रिंशन्मनवः षष्टिरेव च ।
खाद्रयो वसुदस्राः स्युः शीघ्रवृत्तान्यथ क्रमात् ॥२६॥
द्व्यद्रयः खाङ्गनेत्राणि खाब्धयोऽब्ध्यग्निदस्रकाः ।
द्व्यग्नीन्द्वोरवेर्मन्द शुक्रवद् वृत्तमेव च ॥२७॥
एकत्रिंशत् क्षपाभर्त्तुरर्द्धरात्रे विधीयते ।
पातभागाश्च विज्ञेयाः पण्डितैः परिकल्पिताः ॥२८॥
मन्दशीघ्रोच्चयोः क्षेप्यं चक्रार्द्धं बुधशुक्रयोः ।
राशित्रयं तु शेषाणां पात्यते पातसिद्धये ॥२९॥
कुजार्किदेवपूज्यानां भागौ द्वावेव कीर्त्तितौ ।
मन्दपाताश्च शीघ्रोच्चात् सार्द्धांशस्तु भृगुज्ञयोः ॥३०॥
विबुधानां च सर्व्वेषां शीघ्रपाताः प्रकीर्त्तिताः ।
शोधयित्वा क्रमाद्व पातात् विक्षेपांशान् प्रसाधयेत् ॥३१॥
योगविश्लेषनिष्पत्तिरेकानेक-स्वदिग्वशात् ।
विक्षेपः स स्फुटो ज्ञेयः ग्रहस्यैकस्य कीर्त्तितः ॥३२॥

* My attention to the content of this stanza was drawn by Dr. B. B. Dutta, D.Sc.

शन्यस्याप्येवमेव स्यात् शेषाः प्रागुक्तकल्पनाः ।
एतत् सर्व्वं समासेन तन्त्रान्तरमुदाहृतम् ॥३३॥
शीघ्रमन्दोच्चचापार्द्धसंस्कृतात् स्वीयमन्दतः ।
स्फुटमध्यग्रहाः सर्व्वे विशेषः परिकीर्त्तितः ॥३४॥
वेदाग्निरामगुणितान्ययुताहतानि
चन्द्रस्य शून्यरहितान्यथ मण्डलानि ।
स्वैः स्वैर्हतानि भगणैः क्रमशो ग्रहाणां
कक्ष्या भवन्ति खलु योजनमानदृष्ट्या ॥३५॥

Mahābhāskarīya, VII, 21-35.

These stanzas may be thus translated:—

21. "The methods of calculation as set forth in the preceding stanzas are the processes under the *audayika* system; the difference which has to be made in the *ārdharātrika* system is being stated below."

22. "Three hundreds (300) are to be added to the number of civil days in a *Mahāyuga*, the same are to be subtracted from the number of omitted lunar days. From the revolutions of Mercury and Jupiter are respectively subtracted 20 and 4."

23. "Sixteen multiplied by one hundred are the *yojanas* of the earth's diameter; 6480 are the *yojanas* of the sun; 480 of the moon."

24. "The distance of the sun is 689358 *yojanas*; of the moon, the same is spoken of as 51566 *yojanas*."

25. "Sixteen, eight, twenty-four, eleven and twenty-two multiplied by 10 are respectively the longitude of the aphelia of Jupiter, Venus, Saturn, Mars and Mercury."

26. "The epicycles of apsis are of dimensions 32°, 14°, 60°, 70°, and 28° respectively."

27. "The peripheries of the epicycles of conjunction are in the same order, 72°, 260°, 40°, 234°, 132°. The periphery of the sun's epicycle is the same as that of Venus."

28. "Of the moon 31° are the measure of the epicycle of apsis in the *ārdharātrika* system. The longitudes of the nodes are to be taken as the same as they are given by sages in the first system."

29. "To the longitude of the aphelion and to the heliocentric longitudes of Mercury and Venus half a circle is added and three signs (*i.e.*, 90) are subtracted from the node in the case of each of the rest in order to find the apparent node."

30. "In the cases of Mars, Saturn and Jupiter, two degrees are spoken of as corrections from the node as corrected by the equation of apsis; in the cases of Venus and Mercury, a degree and a half represent the correction from their heliocentric positions."

31. "Of all the planets (*Vibudhas*) are thus given the *Śīghras* and nodes; the degrees of celestial latitude are found by subtracting the planets from their nodes."

32. "The determination of the sum or difference is made from the directions, whether the same or opposite. The celestial latitude thus found is to be known as true for a planet."

33. "The same rule holds in another case (*viz.*, finding the true declinations); as to the remaining processes they are the same as in the former system. This all in brief describes the *other tantra* (astronomical treatise)."

34. "The true mean planets are all obtained from the respective aphelia which have been corrected by half the arcs of the *Śīghra* and *Manda* equations; this is another difference."

35. "By 3240000 diminished by one zero, multiply the moon's revolutions; the result divided by the planet's own revolutions, gives the measure in *yojanas* as seen of their orbits."

Now from stanza 21, we gather that 300 is to be added to the number of civil days in a *Mahāyuga*. According to the *Āryabhaṭīya* the number of civil days in this cycle is 1577917500, which increased by 300 becomes 157797800, the number of civil days in a *Mahāyuga* according to the *Khaṇḍakhādyaka*.

Stanza 22 tells us to subtract 20 and 4 respectively from the revolutions of Mercury and Jupiter, and we arrive at the figures 17937000 and 364220 which are the revolutions of Mercury and Jupiter in a *Mahāyuga* according to the *Khaṇḍakhādyaka*.

From stanza 23, we get that—

The Earth's diameter	=	1600 *Yojanas.*
The Sun's diameter	=	6480 ,,
The Moon's diameter	=	480 ,,

This set may be compared with that of the modern *Sūryya-siddhānta* which says that

The Earth's diameter	=	1600 *Yojanas.*	I, 59.
The Sun's ,,	=	6500 ,,	IV, 1.
The Moon's ,,	=	480 ,,	IV, I.

The figures of the *Āryabhaṭīya* are—

The Earth's diameter	=	1,050 *yojanas.*
The Sun's ,,	=	4,410 ,,
The Moon's ,,	=	315 ,,

The next stanza gives the distances of the sun and the moon as 689358 and 51566 *yojanas* respectively. The same figures as worked out by Lalla according to the *Āryabhaṭīya* are 459585 and 34377 in the *Śiṣyadhīvṛddhida*, IV, 3 and 4.

The stanza 25, states the longitudes of the aphelia of planets thus:—

Longitude of	Jupiter's aphelion	=	160°
,, ,,	Venus' ,,	=	80°
,, ,,	Saturn's ,,	=	240°
,, ,,	Mars' ,,	=	110°
,, ,,	Mercury's ,,	=	220°

These agree with the figures of the *Khaṇḍakhādyaka.*

The next stanza states that:—

The periphery of Jupiter's	epicycle of apsis	=	32°
,, ,, ,, Venus'	,, ,, ,,	=	14°
,, ,, ,, Saturn's	,, ,, ,,	=	60°
,, ,, ,, Mar's	,, ,, ,,	=	70°
,, ,, ,, Mercury's	,, ,, ,,	=	28°

These also agree with those given in the *Khaṇḍakhādyaka.*

In stanza 27, are stated the dimensions of the epicycles of conjunction to be—

72° for Jupiter.
260° for Venus.
40° for Saturn.
234° for Mars.
132° for Mercury.

These are the same as in the *Khaṇḍakhādyaka.*

In the next stanza the sun's epicycle is stated to have a periphery of 14° and the moon's epicycle, 31°; the longitudes of the nodes of the planets to be the same as in the *Āryabhaṭīya*. All these are the same in the *Khaṇḍakhādyaka.*

The stanzas 29-32, state some special rules for finding the celestial latitudes of planets. In the next we have a positive assertion of the existence of a तन्त्रान्तर or a separate treatise on astronomy presumably by Āryabhaṭa I.

The next stanza gives rules for finding the geocentric longitudes of planets which may be taken to be the same as in the *Khaṇḍakhādyaka*, II, 18; modern *Sūryasiddhānta*, II, 44; the *Sūryasiddhānta* of Varāha in the *Pañcasiddhāntikā*, XVII, 6; but slightly different from the *Āryabhaṭiya*, *Kālakriyā*, 23-24.

The last stanza gives the dimensions in *yojanas* of the orbits of planets; these are the same as in the modern *Sūryasiddhānta*, XII, 85-89.

We have shown that there is much resemblance in the constants between the *Sūryasiddhānta* of Varāha and the *Khaṇḍakhādyaka* and for the matter of that with the तन्त्रान्तर of Āryabhaṭa I. In my papers "Āryabhaṭa" and "Āryabhaṭa's Lost work," I have established the fact that the *Sūryasiddhānta*, as it existed before the time of Varāha, was made more accurate by him by borrowing the constants from Āryabhaṭa's *ārdharātrika* system. That there was a *Sūryasiddhānta* before the time of Varāha, is seen from section 6 of the table on page xii given before. This point is made clear from another consideration, *viz.*, the star table in the modern *Sūryasiddhānta* which unmistakably points to the conclusion that the longitudes of some stars, *e.g.*, Spica, etc., correspond to a time much anterior to that of Āryabhaṭa I. The great fame of Āryabhaṭa I induced Varāha, the first maker of a *neo-Sūryasiddhānta* to use the elements of Āryabhaṭa's *ārdharātrika* system to supplant the older materials in it. No wonder therefore that there is an opinion in favour of the hypothesis that Āryabhaṭa I was the author of the *Sūryasiddhānta*. If there were a shadow of truth in it, Varāha would have admitted it. Albērūnī indeed says that the *Sūryasiddhānta* was composed by Lāṭa.* We now know that this Lāṭa or Lāṭadeva was one of the first pupils of Āryabhaṭa I. He was the expounder of the *Romaka* and the *Pauliśa siddhāntas*, as we learn from Varāhamihira's *Pañcasiddhāntikā*, i, 3. As Albērūnī's statement is not corroborated by Varāha, we are not inclined to take it as correct. None of the earlier writers suggest that the *Sūryasiddhānta* was in any way modified or changed by Āryabhaṭa I.

It has now been established beyond doubt that the same Āryabhaṭa was the author of the *Āryabhaṭīya* and another *Tantra* which is now lost. There is reason in support of the hypotheses that this *Tantra* itself was the first work of Āryabhaṭa I and that the *Āryabhaṭiya* was the second work from the order in which Varaha mentions them in the stanza quoted before on page xi. If this hypothesis be true the stanza in the *Āryabhaṭiya*—

षष्ठ्यब्दानां षष्टिर्यदा व्यतीतास्त्रयश्च युगपादाः ।
त्र्यधिका विंशतिरब्दास्तदेह मम जन्मनोऽतीताः ॥१०॥

Kālakriyā, 10.

which was translated by me as,

"Now when sixty times sixty years and three quarter *yugas* also have elapsed, twenty increased by three years have elapsed since my birth."

Should now be translated thus:—

"In this *Mahāyuga* when sixty times sixty years and three quarter *yugas* also had passed, twenty increased by three years had elapsed since my birth."

Bhāskara I, the author of the *Mahābhāskariya* and the *Laghubhāskarīya*, wrote a commentary on the *Āryabhaṭīya*.*

The author commenting on this stanza observes that

एतदेवाचार्य्यार्य्यभटस्य शास्त्रव्याख्यानसमये वा पाण्डुरङ्गस्वामि लाटदेव निःशङ्कुप्रभृतिभ्यः प्रोवाच ।

"Or this was addressed by Āryabhaṭa when expounding the science to Pāṇḍuraṅgasvāmī, Lāṭadeva, Niḥśaṅku and other pupils."

This direct pupil of Āryabhaṭa I also says that this stanza does not show that the *Āryabhaṭīya* was composed when Āryabhaṭa I was only 23 years old, but refers to the time when he probably began his career as a teacher of astronomy.

The author of the *Prakāśikā*,† observes

"अस्यायमभिप्रायः । अस्मिन्काले गीतिकोक्तभगणैस्त्रैराशिकेनानीता ग्रहमध्यमोच्चपाताः स्फुटाः स्युः ॥"

The meaning of this stanza is this:—"That at this time, the mean planets, the apogees and the nodes deduced by the rule of three from the planetary revolutions of the *Daśagītikā* would be true."

Hence we are not justified in concluding that the *Āryabhaṭīya* was composed when Āryabhaṭa was only 23 years old. In its present form it was the work of mature age and was done in a "highly finished" ‡ form, the date mentioned in it was the date

* Albērūnī's India, Translation by Sachau, Vol. I, p. 153.

* Manuscript purchased through Dr. Bibhutibhusan Dutta, D.Sc. for the P. G. Lending Library, Calcutta University.

† *Prakāśikā*, the comm. on the *Āryabhaṭīya* by Sūrya Deva Jajvā.

‡ *Pañcasiddhāntikā*, Introduction, p. lvi.

when he became a *guru* or teacher, or the date for which the mean positions would be correct and for subsequent times some corrections were necessary.

Brahmagupta's Originality in the Khaṇḍakhādyaka.

We have already noted in the outline, some points of originality shown by Brahmagupta in the *Khaṇḍakhādyaka*. Some details are here stated. He does not accept the system of Āryabhaṭa's astronomy which he has simplified in the *Khaṇḍakhādyaka* proper as correct. Brahmagupta gives his own corrections to this first part of the work, in the *Uttara Khaṇḍakhādyaka*. In this part he states the longitude of the sun's apogee to be 77° whereas in the *Khaṇḍakhādyaka* proper it is given as 80°. It has been shown in the translation that Brahmagupta is more correct than Āryabhaṭa.* Again Brahmagupta detected that Āryabhaṭa had made the moon's apogee quicker, and nodes slower, than they really are. It has been shown in the translation that Brahmagupta made an over correction in either case.† Again Brahmagupta states that the longitude of Mars's aphelion should be increased by 17° and that of Jupiter by 10°. It has been shown in the translation that Brahmagupta was more correct than Āryabhaṭa.‡ These facts establish the point that the great Indian astronomers from Āryabhaṭa I to Brahmagupta were aware of the methods of separating the two distinct planetary inequalities, *viz.*, that of the apsis and of conjunction in the cases of the five 'star' planets.§ In the *Khaṇḍakhādyaka*, Brahmagupta having given the "sines" and the equations of the sun and the moon at the interval of 15° of arc of the mean anomaly, in the *Uttara Khaṇḍakhādyaka* teaches, for the first time in the history of mathematics, the improved rules for interpolation by using the second difference. This has been detailed in the translation on pages 141-42, and also in the Bulletin of the Calcutta Mathematical Society, Vol. XXIII, No. 3 (1931). In the case when the function is not tabulated at a constant interval, his rule is equally remarkable. Another rule given by Brahmagupta in Chapter VI, stanza 1, is equivalent to the formula

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \text{ in plane trigonometry.}$$

* Translation, p. 139.

† *Ibid*, pp. 140 and 144-46.

‡ *Ibid*, p. 144.

§ *Pañcasiddhāntikā*, Introduction, lii.

Brahmagupta also corrects the dimensions of the epicycle of apsis of the sun and the moon by $-\frac{1}{42}$nd part and $+\frac{1}{48}$th parts respectively.* His correction to Saturn's epicycle of apsis is $-\frac{1}{5}$th part and that to the *Śīghra* epicycle of Mercury $+\frac{1}{16}$th part.† Again in the *Khaṇḍakhādyaka* proper, the treatment of parallax in the calculation of solar eclipses Brahmagupta does not follow Āryabhaṭa. The methods here followed by him are the same as that of the *Brāhmasphuṭasiddhānta*.‡ As has been already remarked these corrections and innovations in the *Uttara Khaṇḍakhādyaka*, paved the way for the acceptance of his great work the *Brāhmasphuṭasiddhānta* as a standard work on astronomy by the western Indian school of astronomers. The directness of the treatment of topics, and the simplicity of calculation taught in the *Khaṇḍakhādyaka* made it a very neat handbook for the beginner. These two works of Brahmagupta were perhaps the only astronomical works in circulation in western India when the Arabs conquered Sind early in the eighth century (712 A.D.) and the new conquerors learnt Indian astronomy and Mathematics from these works as has been observed by Sachau. Alberūnī who came to India early in the 11th century of the Christian era, learnt Indian astronomy chiefly by studying the *Khaṇḍakhādyaka* and the *Bṛhat Saṃhitā* of Varāhamihira, and both of them with the help of the commentary of Bhaṭṭotpala.

Alberūnī and the Khaṇḍakhādyaka.

Sachau in his translation of the *Indika* of Alberūnī, has shown that Alberūnī has made the following references or quotations from the *Khaṇḍakhādyaka* proper and its supplementary or the *Uttara* portion.

(*a*) A reference to the accepted circumference of the earth in the *Khaṇḍakhādyaka* in i, 15 of our translation (Sachau's Albêrûnī, Vol. I, p. 312).

(*b*) The rules for finding the *ahargaṇa* as given in the *Khaṇḍakhādyaka* in i, 3-5 of our translation (Sachau's Alberūnī, Vol. II, 46-47), to which Dr. Schram adds a valuable annotation, the constants being taken from the later *Pauliśa Tantra* as known

* *Ibid*, p. 143.

† *Ibid*, p. 144.

‡ Cf. *Brāhmasphūṭasiddhānta*, xi, 33.

to Bhaṭṭotpala. This Pauliśa astronomy is derived from Āryabhaṭa I's *ārdharātrika* system.*

(*c*) A quotation from the *Uttara Khaṇḍakhādyaka* (Sachau's Albērūnī, Vol. II, pp. 84-86) which in our translation is Chapter X, pp. 148-152.

(*d*) A quotation also probably from the *Uttara Khaṇḍakhādyaka* (Sachau's Albērūnī, Vol. II, p. 87). These stanzas are found in the *Brāhmasphuṭasiddhānta*, XIV, 47-52, also quoted by Bhaṭṭotpala as occurring in the *Brahma Siddhānta* in his commentary on the *Bṛhat Saṁhitā*, IV, 7. The manuscripts which we have used do not show them as occurring in the *Uttara Khaṇḍakhādyaka*. These relate to the dimensions of the *nakṣatras* as seen, as distinguished from the same as calculated.

(*d*) Two quotations from the *Uttara Khaṇḍakhādyaka* relating to the celestial co-ordinates of Canopus and Sirius (Sachau's Albērūnī, Vol. II, p. 91). Our manuscripts do not show these stanzas, which are probably the same as stanzas 35-36 and 40 of Chapter X of the *Brāhmasphuṭa-siddhānta*.

(*e*) Two quotations from the *Khaṇḍakhādyaka* proper as alleged by Albērūnī (Sachau's Albērūnī, Vol. II, p. 116). According to Āmarāja, the first is a couple of stanzas of which the author is Bhaṭṭotpala and not Brahmagupta.† The second quotation cannot be traced. These relate to finding the possibility of an eclipse whether of the sun or of the moon.

(*f*) Two quotations from the *Khaṇḍakhādyaka* proper as asserted by Albērūnī (Sachau's Albērūnī, Vol. II, p. 119). These relate to finding the Lords of the year and of the month. According to Āmarāja the rules in question were given by Bhaṭṭotpala and not by Brahmagupta.‡ Pṛthūdaka in his commentary on the first chapter at its concluding portion says अथाऽत्र खण्डखाद्यके वर्षाधिपमासाधिपानयनमाचार्येणनाभिहितं, *i.e.*, "In this work the *Khaṇḍakhādyaka* the teacher (Brahmagupta) has not given the rules for finding the Lords of the year and the month."

There are, besides these, mention of this work in many other places in Albêrûnî's *Indika*.

* P. C. Sengupta, Āryabhaṭa the Father of Indian Epicycle Astronomy, pp. 33-41, also *Pañcasiddhāntikā*, Introduction, xxxviii.

† *Khaṇḍakhādyaka*, Pt. B. Misra's Edition, p. 145.

§ *Ibid*, pp. 48-49.

Original Contents of the Khaṇḍakhādyaka.

The *Khaṇḍakhādyaka* as composed by Brahmagupta was a work of eleven chapters; the *Khaṇḍakhādyaka* proper consisted of 8 and the *Uttara* part of 3 chapters.

Our translation which follows Pṛthūdaka's text, presents the first eight chapters faithfully, and we have been partially successful in reconstructing the first two chapters only of the supplementary part, which according to our inference had three chapters in all; these were probably

(1) Introductory Corrections and New Methods.

(2) Conjunction of Stars and Planets.

(3) Projection of Eclipses.

One manuscript at our disposal which we have so long taken as following Bhaṭṭotpala's text shows one more chapter, *viz.*, on *Pātas*. If there was really a chapter on this topic, Pṛthūdaka would not have been under the necessity of giving his own rules and illustrations. He has indeed given his rules in no less than twenty-five stanzas of his own, in the concluding part of his commentary on Chapter I.

At any rate we cannot be sure if the *Uttara* or the supplementary part had a chapter on the *Pātas*. The stanzas on this chapter given in the manuscript which we have referred to above, do not read like Brahmagupta's composition.

That there was a chapter on the projection of eclipses we learn from the following evidences from Pṛthūdaka himself. These occur in his commentary at the ends of the Chapters III and IV.

(*a*) एतच्चार्थमुत्तरे सप्रपञ्चं परिलेखोदाहरणं वक्ष्यामि।

(*b*) आदित्यग्रहणं चोत्तरे सप्रपञ्चं सपरिलेखं सोदाहरणम् व्याख्यास्यामः॥

Here he promises to illustrate and explain in detail the projection of lunar and solar eclipses in the supplementary or *Uttara* part. The faulty materials at our disposal have made us give up our attempt at translating the chapter under reference.

Khaṇḍakhādyaka and its Commentators.

Owing to the simplicity of calculation that the *Khaṇḍakhādyaka* taught, it was for a long time, in fact for many centuries, used as a practical handbook for a learner of astronomy in India. Alberuni in his *Indika* early in the eleventh century of the Christian era, noticed that "the cannon *Khaṇḍakhādyaka* is the most universally used among* the Hindus. Of the many commentaries on it written by different writers the most known ones were by Lalla,† Bhaṭṭotpala, Pṛthūdaka, Someśvara, Varuṇa, Āmarāja and others. Of all these commentators, Lalla appears to be the oldest; of Bhaṭṭotpala we know that his time is about 888 of Saka era or 966 A.D.; the times of the remaining authors are not important from the view-point of history As to Lalla the name of his commentary is the *Khaṇḍakhādyapaddhati*.‡ Lalla is again the author of the *Siṣyadhīvṛddhida*; the question is whether the two Lalla's are the same or different persons. We are of opinion, it was the same person who composed the works. The reasons are set forth below:—

(*a*) The first evidence in our favour is obtained by comparing the longitudes of the "Junction stars" given in the *Brāhmasphuṭa-siddhānta* and the *Siṣyadhīvṛddhida*. Some of these longitudes given in either book should be considered as traditional and some corrected by the authors of these two works. For Bentley in his Hindu Astronomy has shown that Brahmagupta's star tables give different values of total precession for 1690 A.D., as obtained from different 'junction stars.'§ Lalla says that his longitudes of 'junction stars' are मुनिभिः प्रकीर्त्तिताः,¶ *i.e.*, "declared by *munis*." Who these *munis* were are not stated at all; many of these longitudes were traditional and some were corrected by Lalla himself. Bentley's investigation of Brahmagupta's table leads to, for the year 1690 A.D., different values of total precession varying from 18° 24′ to 14° 8′. From Brahmagupta's time (628 A.D.) to 1690 A.D., the total shifting of the equinoxes should be about 15°. We would take an error of 1° in his observations and 16° to be the superior limit to the value of the precession, as the criterion for finding which of the longitudes of the junction stars were corrected by him.

* Sachau's Albērūnī, Vol. II, p. 119.

† *Khaṇḍakhādyaka*, Pt. Babua Misra's Edition, p. 1. S. B Dixit, भारतीय ज्योतिःशास्त्र, p. 234.

‡ B. Misra's Edition, p. 27.

§ Bentley's Hindu Astronomy, pp. 83-84.

¶ *Siṣyadhīvṛddhida*, XI, 3.

Judged by this test, his corrections were most probably confined to the cases of:—

Punarvasu	and Lalla's corrections similarly were in cases of	Ārdrā
Maghā		Aśleṣā
P. Phalgunī		Hastā
Citrā (1)		Citrā (1)
Viśākhā		U. Āṣāḍhā (2)
Anurādhā		Abhijit
Jyeṣṭhā		Sravaṇā
P. Āṣāḍhā		Dhaniṣṭhā (3)
U. Āṣāḍhā (2)		P. Bhādrapada (4)
Dhaniṣṭhā (3)		
P. Bhādrapada (4)		
Revatī		

The common stars whose longitudes were corrected by Brahmagupta and Lalla are:—

	Polar long. according to Brahmagupta.	Polar long. according to Lalla.	Excess.
Citrā	183° 0′ 0″	184° 20′ 0″	1° 20′
U. Āṣāḍhā ...	260° 0′ 0″	267° 20′ 0″	
Dhaniṣṭhā ...	290° 0′ 0″	296° 20′ 0″	
P. Bhādrapada ...	326° 0′ 0′	327° 0′ 0″	1° 0′

In the cases of U. Āṣāḍhā and Dhaniṣṭhā, Brahmagupta and Lalla most probably mean different stars. The mean excess comes to 1° 10′, from which Lalla, the author of the *Siṣyadhīvṛddhida* becomes later than Brahmagupta by about 85 years. His date hence comes to about 635 of Saka era or 713 A.D., the date of the *Brāhmasphuṭa-siddhānta* being 550 of Saka era or 628 A.D.

If we consider all the stars of which the polar longitudes as given in Lalla's work are greater than those in Brahmagupta's we are led to a different date for Lalla:

Star.	Polar long. according to Brahmagupta.	Polar long. according to Lalla.	Excess.
Ārdrā	67°	70°	3°
Āśleṣā	108°	114	
Hastā	170°	173°	3°
Citrā	183°	184° 20′	1° 20′
U. Āṣāḍhā	260°	267° 20′	
Abhijit	265°	267°	2° 0′ 0″
Śravaṇā	278°	279° 50′	1° 50′ 0″
Dhaniṣṭhā	290°	296° 20′	
P. Bhādrapada	326°	327° 0′	1° 0 0″
			12° 10′ 0″

Average of the six stars gives a mean excess of 2° 1′ 40″. This would make Lalla's time later than Brahmagupta's by about 140 years. Hence Lalla's time becomes 768 A.D. or 690 of Śaka era. Thus Lalla must have lived between 713 A.D. and 768 A.D.

There is a significant passage in Lalla's great work the *Śiṣyadhīvṛddhida*, which runs as follows:

शाके नखाब्धिरहिते शशिनोऽक्षदस्रैस्तत्तुङ्गतः कृतशिवैस्तमसः षडङ्कैः ।
शैलाब्धिभिः सुरगुरोर्गुणिते सितोच्चात् शोध्यं त्रिपक्षकुहतेऽम्बराक्षिभक्ते ॥
खाङ्केरमाम्बुधिहते क्षितिनन्दनस्य सूर्यात्मजस्य गुणितेऽम्बरलोचनैश्च ।
व्योमाब्धिवेद निहते विदधीतलब्धं शीतांशुसूनुचलतुङ्गकलासु स्विम ॥

This passage occurs twice in I, 59-60 and XIII, 18-19.* It occurs in its proper place in the first chapter which treats of the mean motions and in the thirteenth chapter, where the author gives his genealogy. The corrections which the stanzas give to the mean positions of planets as calculated from its constants are thus expressed:

* *Śiṣyadhīvṛddhida*, Mm. Sudhākara Dvivedi's edition (1886).

"Subtract 420 from the Śaka year elapsed; multiply the remainder severally by 25, 114, 96, 47 and 153 and divide in every case by 250; apply the resulting minutes *negatively* in the following order: to the moon, moon's apogee, moon's node, Jupiter, and the *Śighra* of Venus. Again multiply the same remainder severally by 48, 20 and 420, divide by the same divisor 250; apply the resulting minutes positively in the order—to Mars, Saturn and the *Śīghra* of Mercury."

Here the divisor 250 shows that all these corrections, *viz.*, −25′, −114′, −96′, −47′, −153′, +48′, +20′, and +420′ were found by Lalla 250 years after the time, 420 of Śaka era. Hence Lalla's time is 670 of the Śaka era or 748 of the Christian era. Brahmagupta's time being 628 A.D., Lalla flourished 120 years after him. We are thus led to the conclusion that it was the same person who composed the *Śiṣyadhīvṛddhida* and wrote the commentary *Khaṇḍakhādya-paddhati* on the *Khaṇḍakhādyaka* of Brahmagupta.*

Now the *Khaṇḍakhādyaka* was composed about the year 587 of the Śaka era or 665 A.D. The date of the first commentator, Lalla, has been shown to be 748 A.D., the next commentator, Pṛthūdaka lived about 864 A.D. Bhaṭṭotpala about 966 A.D. and the last commentator lived about 1180 A.D. Thus the *Khaṇḍa-khādyaka* was held in very high esteem for more than six centuries among the distinguished Hindu astronomers and almanac-makers. Taken with proper corrections to the mean positions of planets its rules stand on a par with those of the modern recension of the *Sūryasiddhānta*, as has been demonstrated in the present translation specially in the calculation of eclipses.

Pṛthūdaka's text has been followed throughout in this translation. In our opinion this text gives the stanzas of the work in the order in which they were composed by Brahmagupta. Whenever Pṛthūdaka takes up a stanza from a later chapter to explain a

* Lalla's indebtedness to Brahmagupta is also seen from his rule for finding the instantaneous daily motion of a planet affected by the *Śīghra* equation, in the *Śiṣyadhīvṛddhida*, II, 45-46, as also from the *śīghra* anomalies (II, 47-48) for the stationary points of planets. Both of these are taken from the *Brāhmasphuṭa-siddhānta*, II, 43-44 and 48-49. According to Sudhākara Dvivedi, Lalla's time is 420 of Śaka era and according to Śankara Bālakṛṣṇa Dixit, 560 of the Śaka era. These views are not now tenable in view of our finding. *Cf.* also our paper, "Āryabhaṭa," *loc. cit.*, p. 38.

certain topic, he invariably repeats it in its proper place. In one place he apologises as folllows :—

यदत्राध्यायेऽधिकमूनं व्याख्यातं तद्विद्भिः क्षमनीयम् ।

i.e., "In this chapter whatever has been explained in excess or defect is respectfully requested to be pardoned by the learned."

In Āmarāja's text as also in the text which we have taken as Bhaṭṭotpala's, the stanzas from the *Uttara Khaṇḍakhādyaka* are mixed up with those of the *Khaṇḍakhādyaka* proper. The Berlin manuscript in rotographs, which has been used in preparing this translation breaks up abruptly at the beginning of the *Uttara* part. It has not been possible to partially reconstruct more than two chapters of the *Uttara Khaṇḍkahādyaka* from the faulty materials at our disposal.

To the translation proper have been added three of my papers as appendices. The first two will, it is hoped, bring to prominence the independence of the Hindu astronomers as regards the constants in luni-solar astronomy and as regards the methods of Hindu Spherical Astronomy. The third appendix gives an idea of the planetary motion as it was understood by the great Indian astronomers from Aryabhata I to Bhaskara II.

Much help has been derived, in the work of translating the text, from the edition of the *Brāhmasphuṭa Siddhānta* by the late Mm. Sudhakara Dvivedi for which I express my indebtedness. All references to this earlier work of Brahmagupta in the present translation refer to this learned edition.

P. C. SENGUPTA.

Sexagesimal Units of Time

60 *Bipalas* (*Vipalas*)	=	1 *Pala*, *Bināḍī* (*Vināḍī*), *Bināḍikā* (*Vināḍikā*).
60 *Palas*	=	1 *Ghatikā*, *Nāḍī* or *Nāḍikā*.
60 *Ghaṭikās*	=	1 Day.
6 *Asus*	=	1 *Vināḍī* = 24 Seconds; an *Asu* = 4 Seconds.

Linear Units.

6 husked barley corns in breadth	=	1 *Angulī** (finger-breadth).
24 *Angulīs*	=	1 *Hasta* (cubit)
4 *Hastas*	=	1 Height of man or the bow.
8000 Heights of man †	=	1 *Yojana*.

According to the *Khaṇḍakhādyaka*, the length of the earth's equator = 4800 *yojanas*; hence 1 *yojana* = 5 miles nearly. According to the *Brāhmasphuṭa-siddhānta* ‡ and the modern *Sūryasiddhānta*, a *yojana* is similarly = 5 miles. As measured here by husked barley corns, one *hasta* becomes = 19·5 inches very nearly.

Definitions of Certain Terms.

Akṣadṛkkarma—The process of applying the necessary correction to the 'polar' longitude of a planet to find, at the observer's place, the point of the ecliptic which rises or sets simultaneously with the planet.

Āyanadṛkkarma—The process of applying the necessary correction to the celestial longitude of a planet to convert it into 'polar' longitude.

Śaka era or the era of the Śaka king is now understood to be the era of King Kaniṣka of the Śaka dynasty of Peshawar.

The relationship with the Christian era is this:

Christian era = Śaka era + 78 years.

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* *Brāhmasphuṭa-siddhānta*, XVI, 12; Bhaskara II, however takes one angulī = 8 barley corns in breadth. *Cf.* Albērūnī, Sachau, Vol. II, p. 166.

† *Āryabhaṭīya*, *Daśagītikā*, 7.

‡ *Brāhmasphuṭa-siddhānta*, I, 36. *Sūrya-siddhānta*, I, 59.

Saṅku—The gnomon, a conical solid 12 digits in length or height, the diameter of the base being 2 digits.*

Śighra anomaly—The angle formed by the line joining the earth and sun produced, with the heliocentric radius vector of a planet.

Śighra equation—In the case of an inferior planet it roughly represents the elongation and in the case of a superior planet it is very nearly the annual parallax.

Śighrocca or *Śghra*—In the case of an inferior planet it is the mean heliocentric position; in the case of a superior planet it is the mean position of the sun.

Trigonometrical Functions.

'Sine'—The Indian sine of any arc is defined as the distance of the end of the radius from the 'east-west' or horizontal line.†

'Cosine'—The Indian cosine of an arc is the distance of the end of the radius from the 'north-south' or vertical line.†

'Versed sine'—The Indian versed sine of any arc is the arrow of the double arc lying between the arc and the chord.‡

The translator himself is responsible for the following mistakes occurring in his work and requests his reader to correct them before going over it.

Page 30,	Line 33,	for *vyātipāta*	read *vyatipāta*
,, 31	,, 3	,, Do.	,, Do.
,, 75	,, 9	,, *asus* (=6 sec. of time)	,, *asus* (=4 sec. of time)
,, 112	,, 26	,, opposition	,, conjunction

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* *Brāhmasphuṭa-siddhānta* XXII, 39. *Cf.* Bhāskara II, *Gola*, IX, 9.

† Bhāskara II, *Grahagaṇita*, II, 18-21, Commentary.

‡ Bhāskara II, *Gola*, *Jyotpatti*, 5, also *Gola*, III, 3.

CHAPTER I

On Tithis, Nakṣatras, etc.

1. Having made obeisance to God Mahādeva, who is the Great Cause of this world's Rise (*i.e.*, creation), Existence and Destruction, I shall declare the *Khaṇḍa-khādyaka* (*i.e.*, a short treatise on astronomy, which is as pleasant as food prepared with sugar-candy), which will yield the same results as the great astronomical treatise of Āryabhaṭa.

By Āryabhaṭa, is here meant Āryabhaṭa I, who lived from 476 A.D. The Indian astronomical treatises are divided into three classes: (*i*) *Siddhāntas*, (*ii*) *Tantras*, and (*iii*) *Karaṇas*. The *Siddhāntas* are those of which the calculations start from the beginning of the "creation," the *Tantras* reckon time from the beginning of the *Kaliyuga* or 3102 B.C., while the *Karaṇas* from any subsequent specified time. We shall see that the *Khaṇḍakhādyaka* is a *Karaṇa*.

2. As in most cases calculation by the great work of Āryabhaṭa, for (the knowledge of time and longitude of planets, etc., at) marriage, nativity, and the like, is impracticable for common use every day, this smaller treatise is made so as to yield the same results as that.

This stanza shows the necessity for the present work The next stanza describes the finding of what is known as *ahargaṇa*, or the number of civil days elapsed since the epoch 587 of the Śaka era.

3-5. Deduct 587 from the *Śaka* year, multiply the remainder by 12, to this result add the number of lunar (synodic) months elapsed from the light half of *Caitra*; multiply the sum by 30 and add to it the number of *tithis* elapsed: put down the result increased by 5 separately in two places. In one place divide by 14945; by the quotient diminish the result in the other place, and divide it by 976; by the quotient of intercalary months reduced to lunar days, increase the result in the original place; put down the result below (*i.e.*, in another place), multiply it by 11 and add to it 497; put down the sum below (*i.e.*, in another place) and divide by 111573, diminish it by the quotient obtained from the sum in the first place; divide the new result by 703 and by the quotient of omitted lunar days or *tithis*, diminish the result. The final result is the *ahargaṇa* and begins from Sunday.

We illustrate the above rule from Pṛthūdaka's example:—
Required the *ahargaṇa* in the Śaka year 786 at the end of the 11 *tithis* and one lunar (synodic) month from the light half of *Caitra*.

Process:—

786 − 587 = 199.

Now 199 × 12 + 1 = 2388 + 1 = 2389 total solar months.

Again 2389 × 30 + 11 = 71670 + 11 = 71681 total solar days.

71681 5 14945)71686 4 *da.* 47 *gh.* 47 *pa.*	71681 5 71686 4 *da.* 47 *gh.* 47 *pa.* 71681 *da.* 12 *gh.* 13 *pa.*

Now 71681 *da.* 12 *gh.* 13 *pa.* = 976 *da.* × 73 + 433 *da.* 12 *gh.* 13 *pa.*
Thus the number of intercalary months = 73.

The remainder 433 *da.* 12 *gh.* 13 *pa.* relating to the intercalary months is increased by 17 *ghaṭikās*, for a purpose explained afterwards.

Hence the true remainder relating to intercalary months is taken at 433 *da.* 29 *gh.* 13 *pa.*

Now 73 × 30 = 2190; this is added to 71681 or 71681 + 2190, *i.e.*, 73871 is the total number of *tithis* or lunar days elapsed.

The rule then directs the following operations:

73871 × 11 + 497 which is equal to 813078. This is put down in two places:

I	II
$\frac{813078}{111573}$ = 7 *da.* 17 *gh.* 14 *pa.*	813078 *da.* 7 *da.* 17 *gh.* 14 *pa.* 813070 *da.* 42 *gh.* 46 *pa.* = 703 *da.* × 1156 + 402 *da.* 42 *gh.* 46 *pa.*

Here, in the 2nd place, the remainder 402 *da.* 42 *gh.* 46 *pa.* is increased by 14 *gh.* for a purpose which will be explained afterwards. The true remainder relating to omitted lunar days is thus taken at 402 *da.* 56 *gh.* 46 *pa.*

The quotient 1156 representing the integral number of omitted lunar days is now subtracted from the total lunar days, 73871, and the *ahargaṇa* is now, 73871 − 1156 or 72715.

Now 72715 = 7 × 10387 + 6.

As the remainder is 6, the day at the end of which the *ahargaṇa* is 72715, is Friday.

Before we can take up the explanation of the above rules, we have to remember the meaning of the following terms:

A solar (or *saura*) day = Interval of time in which the sun moves through one degree of longitude; so that 30 solar days = 1 month and 12 months = 1 year.

An intercalary month or an *adhimāsa* = a synodic month which is left out in the adjustment of the lunar calendar to the solar.

A *tithi* = a peculiar time unit in which the moon gains 12° of longitude over the sun. Its mean value is $\frac{1}{30}$th of the synodic month and slightly less than a civil day.

In the stanzas 3-5, the numbers 5 and 497 are additive quantities and need not be considered at present. The rules suggest that for S solar days, the number of intercalary months, I, is given by

$$S\left(1-\frac{1}{14945}\right)\times\frac{1}{976}=I.$$

Now in a *Mahāyuga* of 4320000 years, there are 4320000 × 360 solar days. Hence the number of intercalary months in a *Mahāyuga* by Brahmagupta's rule is

$$(4320000 \times 360)\left(1-\frac{1}{14945}\right) \times \frac{1}{976}$$

$$= \frac{1452556800000}{911645} = 1593336\frac{2280}{911645}.$$

Hence according to the *Khaṇḍakhādyaka* the number of intercalary months is **1593336**.

Now the number of solar days in a *Mahāyuga*

$$= 4320000 \times 12 \times 30 = 1555200000;$$

∴ in 1 *saura* day the number of intercalary months

$$= \frac{1593336}{1555200000} = \frac{1}{976\frac{104064}{1593336}}$$

$$= \frac{1}{976} - x, \text{ suppose.}$$

$$\therefore x = \frac{1}{976} - \frac{1}{976\frac{104064}{1593336}}$$

$$= \frac{1}{976} \times \frac{1}{14945 - \frac{36480}{104064}}$$

$$= \frac{1}{976} \times \frac{1}{14945} \text{ nearly.}$$

$$\therefore \frac{1}{976\frac{104064}{1593336}} = \frac{1}{976} - \frac{1}{976} \times \frac{1}{14945}$$

$$= \frac{1}{976}\left(1-\frac{1}{14945}\right).$$

This shows the method by which Brahmagupta arrived at his rule:*

Again in a *Mahāyuga* of 4320000 years

the number of sun's revolutions = 4320000;

the number of solar months = 51840000.

The number of intercalary months = 1593336;

∴ the number of synodic months = 53433336.

Hence the number of revolutions of the moon

= **57753336**.

The number of *tithis* in a *Mahāyuga* = Synodic months × 30

= 1603000080.

The stanzas next teach us how to find the omitted *tithis* in N number of *tithis*. The rule is:

$$\text{Omitted } tithis \text{ for } N\ tithis = N \times \frac{11}{703}\left(1-\frac{1}{111573}\right).$$

Hence for 1603000080 *tithis* of a *Mahāyuga*, the number of omitted *tithis*

$$= \frac{11 \times 111572 \times 1603000080}{703 \times 111573}$$

$$= \frac{1967349174183360}{78435819}$$

$$= 25082280 - \frac{3960}{78435819}.$$

Thus according to the *Khaṇḍakhādyaka*, the number of omitted *tithis* in a *Mahāyuga* is **25082280**.

Here Brahmagupta simplifies the fraction, and takes,

$$\frac{25082280}{1603000080} = \frac{11}{703}\left(1-\frac{1}{111573}\right).$$

The steps appear to be as follows:—

$$\frac{25082280}{1603000080} = \frac{627057}{40075002}$$

$$\because \frac{1}{63+}\ \frac{1}{1+}\ \frac{1}{10+}\ \frac{1}{14+}\cdots\cdots$$

$$= \frac{11}{703} - x, \text{ suppose,}$$

* *Cf.* the Sanskrit commentary of Āmarāja in Pt. Babuā Miśra's edition.

$$\therefore x = \frac{11}{703} - \frac{627057}{40075002}$$

$$= \frac{11}{703} \times \frac{3951}{440825022} = \frac{11}{703} \times \frac{1}{111573\frac{99}{3951}}$$

$$= \frac{11}{703} \times \frac{1}{111573} \text{ nearly.}$$

Thus $\frac{25082280}{1603000080} = \frac{11}{703}\left(1 - \frac{1}{111573}\right)$

The convergent $\frac{11}{703}$ was known to the author of the *Romaka Siddhānta*.*

We next turn to the additive quantities 5 and 497.

The number of years elapsed since the beginning of the *Kaliyuga* till the end of the 587 of the Saka year

= 3179 + 587 = 3766;

∴ the number of solar days = 3766 × 360
= 1355760.

Now 1355760 ÷ 14945 = 90 *da.* 43 *gh.*

1355760 − 90 *da.* 43 *gh.* = 1355669 *da.* 17 *gh.*

Again 1355669 *da.* 17 *gh.* = 976 *da.* × 1389 + 5 *da.* 17 *gh.*

Here the remainder is 5 *da.* 17 *gh.* Brahmagupta adds these 5 days to the total solar days in finding the *adhimāsas*. These 17 *ghaṭikās* are added to the remainder in the process for finding the intercalary months. The quotient 1389 represents the number of intercalary months in 3766 years

Now 1389 synodic months = 41670 *tithis*.

∴ the total number of *tithis* in 3766 years = 1355760 + 41670
= 1397430.

Now 1397430 × 11 = 15371730

$\frac{15371730}{111573}$ *da.* = 137 *da.* 46 *gh.*

15371730 *da.* − 137 *da.* 46 *gh.* = 15371592 *da.* 14 *gh.*

15371592 *da.* 14 *gh.* = 703 *da.* × 21865 + 497 *da.* 14 *gh.*

* Cf. *Pañca-siddhāntikā*, i. 10; also Sanskrit commentary by Amarāja.

Here the residue is 497 *da.* 14 *gh.* In its place Brahmagupta directs the adding of 497 *da.* in finding the omitted lunar days. The remaining 14 *ghaṭikās* are added to the remainder corresponding to the omitted lunar days.*

Here the total *ahargaṇa* from the beginning of the *Kaliyuga* till 587 of the Saka era,

= 1397430 − 21865
= 1375565.

Now 1375565 = 7 × 196509 + 2.

Hence the 1st day of 587 Saka year elapsed falls on a Sunday, counting a Friday to be the beginning of the *Kaliyuga*.

The remainders are not essential to the finding of the *ahargaṇa*; they are, however, as we shall see later on, used to find the longitudes of the sun and the moon without finding the *ahargaṇa*—a new process by which some tedious calculations are avoided. Again the process of finding the *ahargaṇa* is rather cumbrous. We are here to find the number of civil days corresponding to a certain number of years, months and days; this is best done by taking the length of the year as will be shown below. In practical calculations the synodic months and *tithis* are cumbrous and unnecessary elements; the number of civil days elapsed since the beginning of the year are more easily counted. If, however, the solar calendar is not at all used, as was the case at the beginning and even now in certain parts of India, then the synodic months and *tithis* have to be applied.†

Again in the first steps of the rules, in place of solar months and solar days, the synodic months and *tithis* are added to the solar days. This is an apparently wrong process. But as neither of the remainders in the two divisions (here in this book, there are four) is taken into account in calculating the *ahargaṇa*, this irregularity does not affect the final result.‡

As has been shown above, the number of omitted *tithis* or lunar days in a *Mahāyuga*, according to the *Khaṇḍakhādyaka* is 25082280.

* *Cf.* Amarāja's Sanskrit Commentary.

† As to this method of calculating the *ahargaṇa*, cf. *Pañca-siddhāntikā*, i. 8-10; *Brāhmasphuṭa-siddhānta*, i. 29-30; the modern *Sūrya-siddhānta*, i. 45-61, etc.

‡ Cf. *Siddhānta-siromani*, *Madhyagati-vāsanā*, 15-18 Com. thereon.

Hence the number of civil days in a *Mahāyuga*

= total number of lunar days

− total number of omitted lunar days.

= 1603000080 − 25082280

= 1577917800.

According to the *Āryabhaṭīya*, the number of civil days in a *Mahāyuga* = 1577917500. There is thus a difference of 300 days.* We shall see later on that according to the *Khaṇḍakhādyaka*, the beginning of the astronomical day is not the sunrise at *Laṅkā*.†

Now the length of the year according to the *Khaṇḍakhādyaka* is

$$\frac{1577917800}{4320000} = \frac{292207}{800} \text{ days.}$$

Pṛthūdaka apparently wanted to find the number of integral civil days in 199 years and 1 month.

The total number of days or *ahargaṇa*

$$= 199\frac{1}{12} \times \frac{292207}{800}$$

$$= 199\frac{1}{12} \times 365\frac{1}{4} \text{ nearly}$$

$$= 72684 \cdot 75 + 30 \cdot 43$$

$$= 72715 \cdot 18.$$

Hence the integral number of civil days or *ahargaṇa* is readily seen to be 72715 and is readily checked by considering the day of the week on which the *ahargaṇa* is desired. If the *ahargaṇa* is required for a longer period a closer approximation to the year is necessary.

To find the number of days in 3766 years.

$$\text{The required number of days} = \frac{3766 \times 292207}{800}$$

$$= 1375564 \cdot 45.$$

This has been found before to be 1375565, the difference is made up by considering the day of the week of the last day of the *ahargaṇa*.

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* On this point cf. *Pañca-siddhāntikā*, i. 14; ix. 1; also Introduction to the same, p. xvii; *Brāhmasphuṭa-siddhānta*, x. 5.

† Cf. *Pañca-siddhāntikā*, xv. 20 and the *Brāhmasphuṭa-siddhānta*, x. 13-14.

6. The two remainders relating to the intercalary months and omitted *tithis*, are to be increased respectively by 17 and 14 *ghaṭikās*; the moon's apogee and the node are to be made less by 5 and 10 seconds of arc respectively.

7. The mean Saturn diminished by 3 seconds, the *Śīghrocca* of Mercury diminished by 22 seconds, the mean Mars increased by 2 seconds and the mean Jupiter increased by 4 seconds are equal to the respective mean planets of Āryabhaṭa's "midnight" system.

The next stanza teaches us how to find the sun's mean longitude.

8. From (the *ahargaṇa* found before), multiplied by 800, and increased by 438 and (finally) divided by 292207 are obtained in revolutions, etc., the mean sun, Mercury, and Venus and the *Śīghroccas* of Mars, Jupiter and Saturn.

The rule is equivalent to this:—

$$\text{Mean sun} = \frac{\textit{ahargaṇa} \times 800 + 438}{292207} \text{ revols. etc.}$$

The only figure that requires explanation is 438. This is thus obtained:—The *ahargaṇa* found till the end of 587 of the *Śaka* era is 1375565.

Now $1375565 \times 800 = 292207 \times 3766 + 438$

Hence as the calculations of the *Khaṇḍakhādyaka* really start from the beginning of the *Kaliyuga*, 438 occurs as an additive.

From Pṛthūdaka's example, we have the *ahargaṇa* = 72715.

∴ mean sun at the time mentioned before, *i.e.*, at *ahargaṇa* 72715,

$$= \frac{72715 \times 800 + 438}{292207} = \frac{58172438}{292207},$$

$$= 199 \text{ revols.} + \frac{23245}{292207} \text{ revols.}$$

$$= 199 \text{ revols. } 0 \text{ signs } 28^\circ\ 38'\ 16'',$$

omitting the entire revolutions the mean sun = 0 sign 28° 38′ 16″.

9. The mean sun increased by the number of degrees equal to the *tithis* elapsed multiplied by 12, together with the quotient taken as degrees, of 3 times the remainder relating to the omitted *tithis* divided by ~~137~~ 173, becomes the 'midnight' mean moon.

Now, $\frac{\text{mean moon} - \text{mean sun}}{12^\circ}$

$= \text{Integral } \textit{tithis} \text{ elapsed} + \text{fraction of a } \textit{tithi} \text{ elapsed.}$

$\therefore$ mean moon $=$ mean sun $+ 12^\circ \times$ integral *tithis* elapsed
$+ 12^\circ \times$ fraction of a *tithi* elapsed.

Here the integral *tithis* elapsed are known. The fractional part of a *tithi*, till the end of a given *ahargaṇa*

$= \frac{\text{Remainder relating to the omitted } \textit{tithis} \text{ in finding } \textit{ahargaṇa},}{703}$

expressed in civil days.

Now 703 *tithis* $= (703 - 11)$ or 692 civil days

$\therefore$ 1 civil day $= \frac{703}{692}$ *tithis*.

$\therefore$ the fractional part of a *tithi* required—

$= \frac{\text{Remainder in finding omitted } \textit{tithis}}{703} \times \frac{703}{692}.$

Hence $12^\circ \times$ the required fractional part of a *tithi*

$= \frac{\text{Remainder in finding omitted } \textit{tithis}}{703} \times \frac{703}{692} \times 12^\circ$

$= \frac{3^\circ}{173} \times \text{Remainder in finding omitted } \textit{tithis}.$

Illustration.—In finding the *ahargaṇa*, the elapsed *tithis* were 11, and the remainder in finding the omitted *tithis*, 402 *da*. 56′ 46″.

Here $11 \times 12^\circ = 132^\circ = 4$ signs 12°

and $\frac{402 \; da. \; 56' \; 46'' \times 3^\circ}{173} = 6^\circ \; 59' \; 15''.$

Now the mean sun $= 0$ signs $28^\circ \; 38' \; 16''$;
$\therefore$ the mean moon $= 0$ signs $28^\circ \; 38' \; 16'' + 4$ signs $12^\circ + 6^\circ \; 59' \; 15''$
$= 5$ signs $17^\circ \; 37' \; 31''$.

The next stanza teaches us how to find the mean moon from the *ahargaṇa*.

10. Or, from the *ahargaṇa* multiplied by 600 and increased by $417\frac{1}{5}$ and finally divided by 16393 is obtained in revolutions, etc., the mean moon when it is diminished by the number of minutes from the (same) *ahargaṇa* divided by 4929.

Here the mean moon

$$= \frac{\textit{ahargaṇa} \times 600 + 417\frac{1}{5}}{16393} \text{ revols.} - \frac{\textit{ahargaṇa}}{4929} \text{ min.}$$

As has been shown before (p. 5), the number of moon's revolutions in a *Mahāyuga* of 4320000 years or 1577917800 days, is 57753336. Hence the mean moon in a given *ahargaṇa* from the beginning of the *Kaliyuga*,

$$= \frac{\textit{ahargaṇa} \times 57753336}{1577917800} \text{ revols. etc.}$$

Brahmagupta appears to reduce this process in the following way:—

$$\frac{\textit{ahargaṇa} \times 600}{\frac{1577917800 \times 600}{57753336}}$$

$$= \frac{\textit{ahargaṇa} \times 600}{\frac{946750680000}{57753336}} \text{ revols.}$$

$$= \frac{\textit{ahargaṇa} \times 600}{16393\frac{242952}{57753336}} \text{ revols.}$$

$$= \frac{\textit{ahargaṇa} \times 600}{16393} \text{ revols.} - \left(\frac{\textit{ahargaṇa} \times 600}{16393} - \frac{\textit{ahargaṇa} \times 600}{16393 + \frac{242952}{57753336}} \right) \text{ revols.}$$

$$= \frac{\textit{ahargaṇa} \times 600}{16393} \text{ revols.} - \frac{\textit{ahargaṇa} \times 485904}{2395074675 \cdot 5} \text{ min.}$$

$$= \frac{\textit{ahargaṇa} \times 600}{16393} \text{ revols.} - \frac{\textit{ahargaṇa}}{4929\frac{1}{9}} \text{ min.}$$

Brahmagupta has rejected the fraction $\frac{1}{9}$ in the denominator.

As to the additive quantity $417\frac{1}{3}$ it is obtained in this way :—

The *ahargaṇa* from the beginning of the *Kaliyuga* till the end of 587 of the *Śaka* era,

$$=1375565,$$

and $1375565 \times 600 = 16393 \times 50347 + 629$.

$$\text{Again } \frac{1375565}{4929}\text{ min.} = \frac{\frac{1375565 \times 16393}{4929 \times 360 \times 60}}{16393}\text{ revols.}$$

$$= \frac{211\frac{2}{3}}{16393}\text{ revols. nearly.}$$

Now $629 - 211\frac{2}{3} = 417\frac{1}{3}$ which shows the necessity for the additive quantity. (*Cf.* Āmarāja's commentary.)

Illustration.—The *ahargaṇa* found before is 72715. ∴ the mean moon

$$= \frac{72715 \times 600 + 417\frac{1}{3}}{16393}\text{revols.} - \frac{72715}{4929}\text{ min.}$$

$$= 5 \text{ signs } 17^\circ\ 52'\ 16'' - 14'\ 45''$$

$= 5$ signs $17^\circ\ 37'\ 31''$, which is the same as calculated before.

Pṛthūdaka makes a mistake in his calculation, and doubts if the stanza was really Brahmagupta's. The rule however is quite correct according to the accepted motion of the moon in the *Khaṇḍa-khādyaka.*

The next two stanzas teach the method of finding the mean sun and the mean moon from the processes of finding the *ahargaṇa.*

11-12. Divide the remainder in the finding of omitted lunar days by 692, call the quotient taken as days, etc., the *first*; add it to the remainder in finding the intercalary months : multiply the result by 30 and divide by 1006 and call this new quotient, the *second.* Take the sum of the months (synodic months taken as signs), the days *i.e.*, *tithis* (taken as degrees) and the *first* found before. Put it down in two places, multiply it in the second place by 13. Subtract from each place, the *second*—the two (new) remainders are respectively the mean sun and the mean moon in signs, etc.

Illustration.—The remainder in finding the omitted lunar days $= 402$ *da.* $56'\ 46''$ and that in finding the intercalary months $= 433$ *da.* $29'\ 13''$.

$$\text{Here the } \textit{first} = \frac{402 \textit{ da. } 56'\ 46''}{692} = 0 \textit{ da. } 34'\ 56''$$

$$\text{the } \textit{second} = \frac{(433 \textit{ da. } 29'\ 13'' + 0 \textit{ da. } 34'\ 56'')30}{1006}$$

$$= 12 \textit{ da. } 56'\ 40''.$$

Sum of synodic months and *tithis* from the light-half of *Caitra*

$= 1$ sign 11°.

The sum and the *first* $= 1$ sign 11° + *first*

$= 1$ sign $11^\circ\ 34'\ 56''$.

	1 sign 11° 34′ 56″
The *second*	12° 56′ 40″
Mean sun =	0 sign 28° 38′ 16″

	1 sign 11° 34′ 56″
	13
	6 signs 0° 34′ 0″
The *second*	12° 56′ 40″
Mean moon =	5 signs 17° 38′ 28″

These rules are only approximative. The rationale also is not clear but is connected with the relation between the units—solar months and solar days on the one hand and the units—lunar months (*i.e.*, synodic months) and *tithis*, on the other.

13. The longitude of the sun's apogee is 80°; of the moon, from the *ahargaṇa* from which $453\frac{3}{4}$ has been subtracted and divided by 3232, is obtained in revolutions, etc., the apogee when increased by the minutes of arc from the same *ahargaṇa* divided by 39298.

The longitude of the sun's apogee is thus stated to be 80°. [Cf. *Pañca-siddhāntikā*, ix, 7-8]. The moon's apogee is given by the equation :—

$$\text{Moon's apogee} = \frac{(\textit{ahargaṇa} - 453\frac{3}{4})}{3232}\text{ rev.} + \frac{\textit{ahargaṇa}}{39298}\text{ min.}$$

$-453\frac{3}{4}$ is a *Kṣepa* quantity and we leave it out for the present. In a *Mahāyuga* the total number of civil days $= 1577917800$.

∴ the number of revolutions of the moon's apogee in a *Mahāyuga*

$$= \frac{1577917800}{3232} \text{ revols.} + \frac{1577917800}{39298} \text{ min.}$$

$$= 488217\frac{456}{3232} \text{ revols.} + \frac{1753249}{943152} \text{ revols.}$$

$$= (488217{\cdot}141 + 1{\cdot}859) \text{ revols.}$$

$$= \mathbf{488219} \text{ revols.*}$$

Thus according to the *Khaṇḍakhādyaka*, the revolutions of the moon's apogee in a *Mahāyuga* is taken at **488219**, the same as in the *Āryabhaṭīya* and the *Sūrya-siddhānta* of the *Pañca-siddhāntikā*.

As to the *Kṣepa* quantity, it is obtained as follows:—

The *ahargaṇa* from the beginning of the *Kaliyuga* till the beginning of the epoch of the *Khaṇḍakhādyaka*=1375565. Again according to *Āryabhaṭa* (the *Āryabhaṭīya*, *Daśagītikā.*, 5) $459\frac{3}{4}$ *Mahāyugas* elapsed since the 'creation,' on the first day of the *Kaliyuga*. Hence the longitude of the apogee on that day

$$= \tfrac{3}{4} \times 488219 = 488219 - 122054\tfrac{3}{4}$$
$$= 366154\tfrac{1}{4} \text{ revols.}$$
$$= \tfrac{1}{4} \text{ revol., omitting the entire revolutions.}$$

$\frac{1}{4}$ revolution corresponds to $\frac{1577917800}{488219 \times 4} = \frac{394479450}{488219}$

$$= 808 \text{ civil days nearly.}$$

∴ the *ahargaṇa* for the calculation of the moon's apogee at the epoch of the *Khaṇḍakhādyaka*, should be taken=1375565+808 =1376373.

Now $1376373 = 3232 \times 425 + 2773$.

Again from the *ahargaṇa* 1375565, the minutes of the arc

$$= \frac{1375565}{39298} = \frac{5{\cdot}235}{3232} \text{ revols.}$$

In place of the numerator 5·235, Brahmagupta takes 5·25 or $5\frac{1}{4}$. This $5\frac{1}{4}$ added to the remainder 2773 becomes $2778\frac{1}{4}$; now this subtracted from the first divisor 3232 yields $453\frac{3}{4}$ as the subtractive *Kṣepa* of the stanza.

* *Cf.* the *Āryabhaṭīya*, *Daśagītikā*, 4; *Pañca-siddhāntikā*, ix, 3; also the editor's paper Āryabhaṭa, p. 38.

Illustration.—Let the *ahargaṇa* be, as before, 72715.

The moon's apogee $= \frac{(72715 - 454 + \frac{1}{4})}{3232} \text{ rev.} + \frac{72715}{39298} \text{ min.}$

$= 22$ revols. 4 signs 8° 54′ 5″ + 1′ 51″

$= 4$ signs 8° 55′ 56″ omitting the complete revolutions.

From this 5″ are to be subtracted according to the stanza 6; thus the longitude of the moon's apogee at the required time=4 signs 8° 55′ 51″.

14. Deduct 372 from the *ahargaṇa* and divide it by 6795, the quotient is in revolutions, etc.; add to it the number of degrees obtained by dividing the *ahargaṇa* by 514656; this last result deducted from the whole circle is the longitude of the moon's ascending node.

From this rule the node's negative longitude

$$= \frac{ahargaṇa - 372}{6795} \text{ revols.} + \frac{ahargaṇa}{514656} \text{ degrees.}$$

Now in a *Mahāyuga* the number of civil days=1577917800

∴ the revolution (retrograde) of the node in a *Mahāyuga*

$$= \frac{1577917800}{6795} \text{ revols.} + \frac{1577917800}{514656} \text{ degrees,}$$

leaving out the *Kṣepa* quantity 372.

$= 232217{\cdot}542$ revols. $+ 8{\cdot}519$ revols.

$= 232226{\cdot}061$ revolutions.

Hence according to the *Khaṇḍakhādyaka*, the number of the retrograde revolutions of the moon's node is 232226.* Brahmagupta's rule is not difficult to deduce from this number of revolutions.

In $\frac{3}{4}$ of a *Mahāyuga*, the motion of the moon's node

$= \frac{3}{4} \times 232226$ revols.

$= 174169\frac{1}{2}$ revols.

* Cf. *Āryabhaṭīya*, *Daśagītikā* 4; *Pañca-siddhāntikā*, Introduction, xviii; editor's paper Āryabhaṭa, p. 38.

Now half a revolution $= \dfrac{1577917800}{232226 \times 2}$ civil days

$= \dfrac{6794{\cdot}74}{2}$ civil days

$= 3397$ civil days appxly.

Again the *ahargaṇa* till the epoch of the *Khaṇḍakhādyaka* is 1375565. In the case of the moon's node for the calculation of the *Kṣepa* the *ahargaṇa* should be taken at 1375565 + 3397 or 1378962 = 6795 × 202 + 6372.

Hence from the 1st term of the rule the additive is 6372.

Again from the *ahargaṇa* 1375565 by the second term we get

$$\frac{\dfrac{1375565 \times 6795}{514656 \times 360}}{6795} \text{ revols.} = \frac{50{\cdot}4488}{6795} \text{ revols.}$$

Here instead of 50·4488, Brahmagupta takes 51. Now 51 + 6372 = 6423 and the 1st divisor 6795 − 6423 = 372 which is the negative additive here.

Illustration.—Let the *ahargaṇa* be 72715 as before; hence the negative longitude of the moon's ascending node,

$= \dfrac{72715-372}{6795}$ rev. $+ \dfrac{72715}{514656}$ degrees

$= 10$ rev. 7 signs 22° 44′ 30″ + 0° 8′ 28″

$=$ 7 signs 22° 52′ 58″, omitting the complete revolutions.

∴ the longitude of the moon's ascending node = 4 signs 7° 7′ 2″.

From this longitude 10″ according to the stanza 6, and 96′ also according to Pṛthūdaka and his school, must be subtracted. Thus according to Pṛthūdaka the longitude of the moon's ascending node at the time,

$=$ 4 signs 5° 30′ 52″.

Thus from the *ahargaṇa* 72715, Pṛthūdaka calculates:—

The mean longitude of the sun = 0 sign 28° 38′ 16″.

................................moon = 5 signs 17° 37′ 31″.

....................moon's apogee = 4 signs 8° 55′ 56″.

.................moon's asc. node = 4 signs 5° 30′ 52″.

These are the mean longitudes on Friday *midnight* at Ujjayinī. If the longitudes are needed for a particular time and place, the necessary corrections for time and longitude must be made. These can not be effected without finding the daily motions. Pṛthūdaka, then taking the *ahargaṇa* to be 1, calculates by the foregoing rules, the mean daily motions of the sun, etc., to be as follows:—

Sun	Moon	Moon's apogee	Moon's nodes (retrograde)
59′	790′	6′	3′
8″	34″	40″	11″

For details of his calculation see the Sanskrit commentary.

Pṛthūdaka then says that by subtracting the entire mean motion from the mean longitude, the planet is got for the previous midnight—now by interpolating for the half night and the preceding day, the planet is obtained for the sunrise, etc. He then speaks of the corrections for the longitude of the station and says that the (meridian) line passing through Laṅkā and the north pole is the line for which the longitudes are found from the rules. He quotes a stanza current among the Pauliśa school of astronomers, which may be thus translated. "On the cities, Ujjayinī (Ojein), Rohītaka (Rohtak), Kurukṣetra, the Himalayas and the poles no correction for difference of longitude is to be made as these places are on the (prime) meridian." Pṛthūdaka most solemnly asserts that the above stanza could not have been composed by Puliśa. While residing at Kurukṣetra, he found that the difference of longitude from Ujjayinī was $1\frac{1}{2}$ *ghaṭikās*, E, *i.e.*, 36 min. E,—an impossible result, which he obtained by the method of a celestial signal, *viz.*, the difference in the beginning of a lunar eclipse, as calculated and as observed. He never suspected the possibility of the following pitfalls—(*i*) that his calculation and his data for it might be wrong; (*ii*) that there might be errors of observation. A *sannyāsin*, as he was, he might check his result by the rougher method of *Brahma-sphuṭa-siddhānta* (i, 36-38) or of the *Pañca-siddhāntikā* (iii, 13-14). In the latter work he might have seen it stated that the difference in longitude of Benares from Ujjayinī was 1 *ghaṭikā* 40 *palas*. One might think Pṛthūdaka to be of sufficient mathematical acumen to be able to devise the method of Bhāskara in his *Gola*, *Praśnādhyāya*, 28. Pṛthūdaka, however, sticks to his result $1\frac{1}{2}$ *ghaṭikās* as the difference of longitude between Kurukṣetra and Ujjayinī. This difference of longitude as used by the *Khaṇḍakhādyaka* is taken to be equal to the length of the equator corresponding to the difference in time. The length of the equator is taken at 4800 *yojanas*. Hence the difference in longitude at Kurukṣetra $= \frac{1\frac{1}{2} \times 4800}{60}$ $= 120$ *yojanas*, according to Pṛthūdaka Swāmī.

15. Multiply the difference in longitude (from Ujjayinī), by the (mean) daily motion of a planet (in minutes) and divide by 4800; apply the quotient taken as minutes negatively in places east of the meridian line of Ujjayinī and positively in places lying west.

As explained before, the difference in longitude is measured in the *Khaṇḍakhādyaka* in *yojanas*. At Kurukṣetra where Pṛthūdaka was staying at the time of writing the commentary, this difference in longitude as taken by him = 120 *yojanas* E.

Now the sun's mean motion = 59′ 8″.

Correction for difference in longitude

$$= \frac{59' \; 8'' \times 120}{4800}$$

$$= 1' \; 28''.$$

The sun's longitude as found before = 0 sign 28° 38′ 16″.

∴ the sun's longitude for the midnight at Kurukṣetra

= 0 sign 28° 36′ 48″.

Pṛthūdaka then states the mean motions thus:—

Sun	Moon	Mars	Mercury	Jupiter	Venus	Saturn	Moon's apogee	Moon's asc. node
59′ 8″	790′ 34″	31′ 26″	245′ 32″	4′ 59″	96′ 7″	2′ 0″	6′ 40″	3′ 10″

and then gives the following corrections for the difference in longitude at Kurukṣetra:—

Sun	Moon	Moon's apogee	Moon's asc. node	Mars	Mercury	Jupiter	Venus	Saturn
1′ 28″	19′ 45″	0′ 10″	0′ 4″	0′ 47″	6′ 8″	0′ 7″	2′ 24″	0′ 3″

In the next stanza, Brahmagupta gives the sun's equations at intervals of 15°.

16. 35′, 67′, 95′, 116′, 129′, 134′ are the sun's equations for every half sign (of mean anomaly).

Here the sun's maximum equation of the centre is 134′. If P stand for the circumference of the sun's epicycle of apsis, then

$$\frac{P^\circ \times 3438'}{360^\circ} = 134' \quad \therefore \quad P = 14^\circ$$

Now taking P = 14° and the formula for the sun's equation to be

$$\frac{P^\circ \times 3438' \times \sin \text{(mean anomaly)}}{360^\circ}$$

$$= \frac{1337'}{10} \sin \text{(mean anomaly)}.$$

The calculated equations may be tabulated as follows:—

Mean anomaly ...	0°	15°	30°	45°	60°	75°	90°
Equations ...	0	34′·9	66′·8	94′·5	115′·8	129′	133′7

Hence the equations are in fair agreement with those given by Brahmagupta, and the circumference of the sun's epicycle is 14° according to *Khaṇḍakhādyaka*.

Illustration.—The sun's mean longitude at Kurukṣetra is 0 sign 28° 36′ 48″, and the sun's apogee is at 80° or 2 signs 20°.

Now mean anomaly

= Mean sun — long. of apogee,

= 0 sign 28° 36′ 48″ — 2 signs 20°,

= 10 signs 8° 36′ 48″ which is greater than 270°.

Hence the equation must be found from 12 signs — 10 signs 8° 36′ 48″ or 1 sign 21° 23′ 12″ or 3083′·2. Now 3083′·2=900′ × 3 + 383′·2, hence as the equations are tabulated at intervals of 15° or 900′, the required equation lies between the 3rd and the 4th figures or between 95′ and 116′. Here the difference for 900′=21′.

$$\therefore \text{ the reqd. equation } = 95' + \frac{21' \times 383\cdot 2}{900}$$

$$= 95' + 8'\ 56''$$

$$= 1°43'\ 56''.$$

Here as the mean anomaly is greater than 6 signs or 180°, the equation is positive.

∴ the sun's apparent longitude

= 0 sign 28° 36′ 48″ + 1° 43′ 56″

= 1 sign 0° 20′ 44″.

The next stanza gives the moon's equations.

17. The moon's (corresponding equations) are 77′, 148′, 209′, 256′, 286′, 296′. The (mean) planet diminished by the apogee is the *Kendra* or the mean anomaly.

Here the moon's equation of apsis for 90° of mean anomaly is stated to be 296′. If P_1 be the circumference in degrees of the moon's epicycle of apsis, then

$$\frac{P_1 \times 3438'}{360} = 296'$$

$$\therefore P_1 = \frac{296}{3438} \times 360° = \mathbf{31°} \text{ nearly.}$$

By taking 31° to be the circumference of the epicycle of apsis of the moon, the corresponding equations are as follows:—

Degrees of anomaly...	0°	15°	30°	45°	60°	75°	90°
Equations ...	0′	76′·6	148′	209′·3	256′·37	285′·46	296′·05

Thus according to the *Khaṇḍakhādyaka*, the periphery of the moon's epicycle of apsis is **31°**.*

* Cf. *Pañca-siddhāntikā*, ix, 7-8; the editor's paper Āryabhaṭa, p. 39.

Illustration.—The moon's mean longitude at Kurukṣetra.

= 5 signs 17° 17′ 46″.

The long. of apogee = 4 signs 8° 55′ 46″.

∴ the mean anomaly = 1 sign 8° 22′ 0″,

= 2302′ = 900 × 2 + 502′.

Thus the equation lies between 148′ and 209′.

Now the difference for 900′ or 15° = 61′

$$\therefore \text{ the equation } = 148' + \frac{61' \times 502}{900}$$

$$= 148' + 34'\ 1'' = 3°\ 2'\ 1''.$$

Here as the mean anomaly is less than 180°, the equation is to be applied negatively.

∴ the moon's apparent longitude

= 5 signs 17° 17′ 46″—3° 2′ 1″

= 5 signs 14° 15′ 45″.

To this is applied the *Bhujāntara* correction as explained in the next stanza. In the case of the moon it is $\frac{1}{27}$th of the sun's equation and in this particular case it is $\frac{1}{27}$ of 103′ 56″ = 3′ 55″. This is applied positively to the moon as the equation is done in the case of the sun.

Thus the moon's apparent longitude is taken

= 5 signs 14° 19′ 40″.

The next stanza teaches where the equation is positive or negative and the application of $\frac{1}{27}$ of sun's equation to the moon.

18. In odd quadrants from the arc passed over and in even quadrants from the arc to be passed over, is obtained the corresponding equation, which is applied negatively or positively according as the mean anomaly is less or greater than six signs or 180°. Of the moon the further equation is $\frac{1}{27}$ of the sun's equation (applied negatively or positively in accordance with the sun's mean anomaly).

A full explanation of the rationale of the above three stanzas calls for a regular exposition of the epicyclic astronomy, which we mean to do in the appendix. The Sanskrit texts are the following:—(i) *Āryabhaṭīya*, *Kālakriyā*, 17-22; (ii) *Brāhmasphuṭa-siddhānta*, *Gola*, 24-30; (iii) Bhāskara's *Golādhyāya*, *Sphuṭa-gati-vāsanā*, 7, 10-37. The references in English are the following:— (i) Burgess's translation of the *Sūrya-siddhānta*, II, 34-46; (ii) Translation of the *Siddhānta-śiromaṇi* by Wilkinson and Bāpudev Śāstri; (iii) the present editor's translation of the *Āryabhaṭīya*, Calcutta University Journal of the Department of Letters, Vol. XVI, pp. 35-39; also his Papers on Hindu Mathematics and Astronomy, pp. 27-45.

This (18th) stanza says that the equation of the centre is negative from apogee to perigee and that it is positive from perigee to apogee. The direction for working out the equation for a given value of mean anomaly is evident; as the equation is a sine function, it is to be obtained like a sine function. As to the $\frac{1}{27}$th of the sun's equation to be applied to the moon, it is obtained as follows:— At the end of the mean *ahargaṇa* or 72715 civil days, the *mean sun on the ecliptic* is supposed to be at its lower transit: the apparent sun is ahead (or behind) it by the sun's equation of the centre. The time taken by the celestial sphere to turn through the sun's equation of the centre

$$= \frac{\text{Sun's equation expressed in minutes}}{21600'}\ da.$$

The moon's motion in this fraction of a day

$$= \frac{\text{Sun's equation} \times 790'\ 34''}{21600'} = \frac{1}{27} \text{ of the sun's equation.}$$

This is to be applied according to the value of the sun's mean anomaly. The next stanza teaches us how to rectify the mean daily motions of the sun and the moon.

19. Divide by 15, the tabular difference of the sun's equations, which corresponds to the mean anomaly, and divide by 8 the moon's tabular difference multiplied by 7; the results are to be applied to their mean motions, negatively, positively, positively and negatively in the four quadrants of the mean anomaly.

Sun's equations ...	0′	35′	67′	95′	116′	129′	134′
Tab. diff. of equations ...	35′	32′	28′	21′	13′	5′	
Corrections to mean motion ...	2′ 20″	2′ 8″	1′ 52″	1′ 24″	0′ 52″	0′ 20″	

The *rationale* appears to be this:—

The sun's longitude, l, after t days from a given date is given by $l = nt - E$, where n stands for the sun's mean motion and E, the equation of the centre; after one day the longitude will be given by

$l' = (t+1)n - (E + \text{increase of } E$ for an increase of n in the mean anomaly),

$$\text{or } l' = n(t+1) - (E + \frac{\text{Tab. diff. of equations} \times n}{900}.$$

$$\therefore \text{ daily motion} = n - \frac{\text{Tab. diff. of equations} \times n}{900}.$$

Roughly $n = 60'$ in the case of the sun, and $\frac{60}{900} = \frac{1}{15}$ which explains the division by 15 in the case of the sun's tabular differences of equations.

In the case of the moon, the equation giving the longitude is,

$l = nt - E$, where n is the moon's mean daily motion in minutes; let n' be the motion of the moon's apogee, then the increase of the mean anomaly per day is $(n - n')$. After one day the longitude l' will be given by

$l' = n(t+1) - \{E + \text{increase of } E \text{ for } (n-n') \text{ increase of mean anomaly}\}$;

$$\therefore\ l' = n(t+1) - \left\{E + \frac{(n-n') \text{ Tab. diff. of Equations}}{900}\right\}$$

when n and n' are expressed in minutes.

Daily motion $= l' - l$

$$= n - \frac{(n-n') \text{ Tab. diff. of Eqns.}}{900}$$

In the case of the moon, $n = 790'\ 34''$, $n' = 6'\ 40''$;

$\therefore\ n - n' = 783'\ 54'' = 783'{\cdot}9$.

Now $\frac{783 \cdot 9}{900} = \frac{871}{1000} = \frac{1}{1+}\,\frac{1}{6+}\,\frac{1}{1+}\,\frac{1}{3+}\,\frac{1}{32}$

The convergents are $\frac{1}{1}, \frac{6}{7}, \frac{7}{8}$ etc.

Brahmagupta uses the convergent $\frac{7}{8}$, which explains the multiplier 7 and divisor 8.

Moon's equations ...	77′	148′	209′	256′	286′	296′
Tabular difference ...	77′	71′	61′	47′	30′	10′
Corrections to moon's mean motion ...	67′ 22″	62′ 7″	53′ 22″	41′ 7″	26′ 15″	8′ 45″

Brahmagupta is not quite satisfied with the above rough rule, and in the next gives his complete rule.

20. Multiply the motion in anomaly by the tabular difference of equations (at the mean anomaly), and divide by 900; the results are the corrections for the sun and the moon's daily motions. In case of Venus, etc., follow the same rule and as stated before, *i.e.*, apply the results to the respective mean motions, negatively, positively, positively and negatively.

The rationale of this stanza has already been explained. In the increasing state of the equations, the corrections to the mean motions have the same sign as the equations, and in the decreasing state the opposite sign. The equations increase in the first quadrant, decrease in the second, increase in the third and decrease in the fourth. Hence the corrections are −, +, +, and − in the four quadrants of the mean anomaly.

By the rules of the stanza, the corrections to the mean motions become:—

To sun's mean motion ...	2′ 18″	2′ 6″	1′ 50″	1′ 22″	1′ 8″	0′ 20″
To moon's mean motion ...	67′ 9″	61′ 55″	53′ 12″	40′ 59″	26′ 10″	8′ 43″

The sun's longitude as found before = 1 sign 0° 20′ 44″.
The moon's longitude = 5 signs 14° 19′ 40″.
The sun's mean anomaly = 10 signs 8° 36′ 48″.
Tabular difference of equations = 21′,
∴ the correction to the sun's mean motion = 1′ 22″.
∴ the sun's rectified daily motion = 59′ 8″ − 1′ 22″ = 57′ 46″.
The moon's mean anomaly = 1 sign 8° 22′;
∴ Tabular difference of equations = 61′,
the moon's rectified daily motion = 790′ 34″ − 53′ 12″ = 737′ 22″.

Pṛthūdaka wanted to illustrate the finding of *nakṣatra, tithi, karaṇa*, etc., from the longitudes of the sun and the moon. For this purpose it is necessary to find the length of the day and night; this leads to the finding of ascensional difference or half the variation of the day from 30 *ghaṭikās* or 12 hours. These topics are given in the section called *Tripraśnādhikāra* (*i.e.*, finding the meridian, latitude, and local time at the place of the observer). But as they are necessary in this connection, they are here considered by Pṛthūdaka and we also follow him. The ascensional differences are first considered.

21. 159 divided by 16, 65 divided by 8, 10 divided by 3, each multiplied by the equinoctial shadow are (the tabular differences of) ascensional difference expressed in *binādīs*.

The formula in the *siddhāntas* for the 'sine' of the ascensional difference is given as—R sin (ascensional difference)

$$= \frac{R \sin \delta \times \text{Equinoctial shadow}}{12} \times \frac{R}{R \cos \delta},$$ where δ is the sun's declination, and the equinoctial shadow $= 12 \tan \phi$, ϕ being the latitude of the station.*

Or, R Sin (ascensional difference) $= \frac{\tan \delta \times R}{12} \times$ Equinoctial shadow,

where R = 3438′, the measure of the radian according to Āryabhaṭa. If the obliquity of the ecliptic be 24°, the value of δ for the longitude of the sun 30°, is $\sin^{-1}(\sin 30° \times \sin 24°)$; for longitude 60°, it is $\sin^{-1}(\sin 60° \times \sin 24°)$; for longitude 90°, it is 24°.

* Cf. *Brāhmasphuṭa-siddhānta*, ii. 57-58; *Surya-siddhānta*, ii. 61; etc.

Now $\delta_1 = \sin^{-1}(\sin 30° \times \sin 24°) = 11° 44' 2''$.

$\delta_2 = \sin^{-1}(\sin 60° \times \sin 24°) = 20° 37' 43''$.

$\delta_3 = 24°$.

$$\therefore \frac{\tan \delta_1 \times 3438'}{12} = 59'{\cdot}508 = \frac{59{\cdot}508}{6} \textit{ binādikās}$$

$$= \frac{158{\cdot}688}{16} \textit{ binādikās}$$

$$= \frac{159}{16} \textit{ binādikās approximately.}$$

Now $\frac{\tan \delta_2 \times 3438'}{12} = 107'{\cdot}8516$.

$$\therefore \frac{\tan \delta_2 \times 3438'}{12} - \frac{\tan \delta_1 \times 3438'}{12}$$

$$= 107'{\cdot}8516 - 59'{\cdot}5080$$

$$= 48'{\cdot}3436 = \frac{48{\cdot}3436}{6} \textit{ bin.}$$

$$= \frac{64{\cdot}4581}{8} \textit{ binādikās.}$$

$$= \frac{65}{8} \textit{ binādīs according to Brahmagupta.}$$

Again $\frac{\tan \delta_3 \times 3438'}{12} = 127'{\cdot}558$ and

$$127'{\cdot}558 - 107'{\cdot}8516 = 19'{\cdot}7064 = \frac{19{\cdot}7064}{6} \textit{ bin.}$$

$$= \frac{9{\cdot}8532}{3} \textit{ binādīs} = \frac{10}{3} \text{ according to Brahmagupta.}$$

These figures of Brahmagupta, *viz.*, $\frac{159}{16}, \frac{65}{8}, \frac{10}{3}$, are comparable with 10, 8, and $3\frac{1}{3}$ of Bhāskara. *Grahagaṇita, Spaṣṭādhikāra*, 50-51. The *Pañca siddhāntikā* figures are 10 8·25, 3·375, ii. 10-12.

The tabular differences are thus:

$\frac{159}{16} \times$ Equinoctial shadow,

$\frac{65}{8} \times$ Equinoctial shadow,

$\frac{10}{3} \times$ Equinoctial shadow.

According to Pṛthūdaka at Kurukṣetra the equinoctial shadow = 7. At that place the tabular differences of the ascensional differences are,

$$\frac{159 \times 7}{16} = \frac{1113}{16} = 69{\cdot}57 \textit{ binādīs.}$$

$$\frac{65 \times 7}{8} = \frac{455}{8} = 56{\cdot}87 \textit{ binādīs}$$

$$\frac{10 \times 7}{3} = \frac{70}{3} = 23{\cdot}33 \textit{ binādīs.}$$

Pṛthūdaka takes these to be 69, 57, and 23. The sun's longitude as found before = 1 sign 0° 20′ 44″. As one sign is passed over by the sun, the ascensional difference

$$= \left(69 + \frac{57 \times 21}{3600}\right) \textit{binādīs} = \left(69 + \frac{1071}{3600}\right) \textit{binādīs.}$$

= 69 *binādīs* approximately

= 1 *ghaṭikā* 9 *binādīs*.

∴ half the day = 16 *ghaṭikās* 9 *binādīs*

Half the night = 13 *ghaṭikās* 51 *binādīs*

22. Multiply the daily motion in minutes by the ascensional difference and divide by 3600 (*i.e.*, number of *binādikās* in a whole day), subtract the result from the planet for sunrise and add it to the planet for sunset when the sun is in the northern hemisphere; do the reverse when the sun is in the southern hemisphere.

In the *Khaṇḍakhādyaka*, the planets calculated, are for the apparent midnight; the planets for the sunrise are first obtained by subtracting ¼th of the daily motion from the midnight longitude, and planets for sunset by subtracting ¼th of the daily motion from

... longitude. The longitudes for sunrise and sunset are ... time when the sun is on the 6 o'clock circle, either in the morning or in the evening. The corrections spoken of in this stanza make the longitudes true for the apparent sunrise. No illustration is necessary.

23. Fifteen, respectively diminished and increased by the ascensional difference when the sun is in the northern hemisphere, and respectively increased and decreased when the sun is in the southern hemisphere, doubled will give the lengths of the night and the day in *ghaṭikās.*

As found before, half the night	= 13 *ghaṭikās* 51 *bināḍis.*
∴ the whole night	= 27 *ghaṭikās* 42 *bināḍis.*
Half the day	= 16 *ghaṭikās* 9 *bināḍis.*
∴ the whole day	= 32 *ghaṭikās* 18 *bināḍīs.*

The next stanza teaches how to find the *nakṣatra* on any day.

24. If of any planet the longitude in minutes be divided by 800, the quotient gives the *nakṣatra* passed over from *Aswinī*; of the current *nakṣatra*, the portions elapsed and to be passed over divided by the daily motion give the days, and then the remainder multiplied by 60 and divided as before gives the *ghaṭikās*, respectively elapsed and to be passed over, of the current *nakṣatra.*

A *nakṣatra* = $\frac{1}{27}$ of the whole circle

$$= \frac{360 \times 60'}{27} = 800' \text{ of the arc.}$$

Illustration.—The longitude of the moon as found before = 5 signs 14° 19′ 40″ and the moon's daily motion = 737′ 22″.

Now 5 signs 14° 19′ 40″ = 9859′ 40″,

and 9859′ 40″ = 800′ × 12 + 259′ 40″.

Hence 12 *nakṣatras* have been passed over at the midnight of the day and of the 13th *nakṣatra* 259′ 40″ have also been passed over.

Now 800′ − 259′ 40″ = 540′ 20″,

Thus 540′ 20″ are to be passed over, of the current *nakṣatra.*

Now $\frac{259' \, 40''}{737' \, 22''} \times 60$ *ghaṭikās* = 21 *ghaṭikās* 8 *bināḍīs*

and $\frac{540' \, 20'' \times 60}{737' \, 22''}$ *ghaṭikās* = 43 *ghaṭikās* 58 *bināḍīs.*

Now the length of the day + half the night

= 32 *gh.* 18 *bin.* + 13 *gh.* 51 *bin.* = 46 *ghaṭikās* 9 *bināḍīs*

∴ the time elapsed since the previous sunrise when the current *nakṣatra* began = 46 *gh.* 9 *bin.* − 21 *gh.* 8 *bin.*

= 25 *gh.* 1 *bin.*

Again the time after the next sunrise up to which the current *nakṣatra* continues

= 43 *gh.* 58 *bin.* − half the night

= 43 *gh.* 58 *bin.* − 13 *gh.* 51 *bin.*

= 30 *gh.* 7 *bin.*

25. The moon, diminished by the sun and reduced to minutes and then divided by 720, yields the number of *tithis* passed over. From the parts elapsed and to be passed over, multiplied by 60 and divided by the difference of the daily motions of the moon and the sun, are obtained the *ghaṭikās* elapsed and to be passed over, of the current *tithi.*

As defined before a *tithi* is a peculiar time unit in which the moon gains 12° or 720′ of longitude over the sun and there are 30 *tithis* from conjunction to conjunction. As found before:—

The moon's longitude	= 5 signs 14° 19′ 40″.
The sun's longitude	= 1 sign 0° 20′ 44″.
The moon's daily motion	= 737′ 22″.
The sun's daily motion	= 57′ 46″.
Moon − Sun	= 4 signs 13° 58′ 56″ = 8038′ 56″
	= 720′ × 11 + 118′ 56″.

Hence at this time 11 *tithis* have passed; of the 12th *tithi* 118′ 56″ have also passed and 601′ 4″ yet remain.

The difference of daily motions = 679′ 36″.

$$\text{Now } \frac{118' \, 56'' \times 60}{679' 36''} = 10 \; gh. \; 30 \; bin.$$

$$\frac{601' \, 4'' \times 60}{679' \, 36''} = 52 \; gh. \; 57 \; bin.$$

Hence the time elapsed since the preceding sunrise when the 11th *tithi* ended = 46 *gh.* 9 *bin.* − 10 *gh.* 30 *bin.*
= 35 *gh.* 39 *bin.*

Also the time after the next sunrise when the 12th *tithi* will end
= 52 *gh.* 57 *bin.* − 13 *gh.* 51 *bin.*
= 39 *gh.* 6 *bin.*

The next stanza speaks of the fixed *karaṇas*. *Karaṇa* = ½ of a *tithi*.

26. The second half of the 14th *tithi* of the dark half of the month is called *śakuni karaṇa*; the first half of the 15th, *catuṣpada karaṇa*, the second half is called *nāga karaṇa* and the first half of the first *tithi* of the light half is called *kingstughna*.

After *kingstughna*, come the seven *karaṇas*, which are named *Vava*, *Vālava*, *Kaulava*, *Taitila*, *Gara*, *Vaṇij*, and *Viṣṭi*. These movable *karaṇas* or ½ *tithis*, make 8 complete cycles up to the first half of the 14th *tithi* of the dark half of the month, then we have the 4 fixed *karaṇas* as stated in this stanza. A *karaṇa* represents the time in which the moon gains 6° of longitude over the sun. The next stanza gives the rule for finding the movable *karaṇas*.

27. Divide the moon diminished by the sun and reduced to minutes by 360, lessen the quotient by one and divide by 7, from the remainder are (to be counted) the *karaṇas* beginning with *vava*. The rest (*i.e.*, the remaining processes) are like those of *tithis*.

Here, moon − sun = 8038′ 56″ = 360′ × 22 + 118′ 56″

∴ the quotient here is 22, lessened by 1 and divided by 7 yields 0 or 7 as the remainder, hence the *karaṇa* that is over is *Viṣṭi*. The current *karaṇa* is *vava*, which will last till 16 *gh.* 52 *bin.* after midnight or till 3 *gh.* 1 *bin.* after the next sunrise.

28. When the sum of the sun and the moon is equal to half a circle or the whole, it is respectively called *vyatipāta* or *vaidhṛta*: the days (whether elapsed or to come) are obtained from the excess or defect of the sum (of the sun and the moon) from 6 signs or the whole circle, divided by the sum of their daily motions. The *pāta* whether *vyatipāta* or *vaidhṛta* takes place when the sun and the moon have the same declinations (numerically).

The process of finding the time when the sum of the longitudes of the sun and the moon is equal to 180° or 360°, presents no great difficulty, the rule is sufficiently clear. But to find the time when they have numerically the same declinations is a matter involving somewhat tedious calculations, if the declinations of the sun and the moon are not daily found and tabulated. The rule for finding the declinations is given by Brahmagupta in the *Tripraśnādhikāra* but as it is necessary here, Pṛthūdaka introduces it and we also follow him.

29. The declinations (for each half sign) in minutes are 362, 703, 1002, 1238, 1388, 1440, increased or decreased by the planet's celestial latitude (according as they have the same or opposite denominations).

Taking 24° to be the obliquity of the ecliptic according to the *siddhāntas* the declinations at intervals of half a sign work out to be

363′, 704′, 1003′, 1238′, 1388′, 1440′,

while 362′, 703′, 1002′, 1238′, 1388′, 1440′,

are Brahmagupta's figures which show a slight discrepancy of 1′ in the first three values. The spherical astronomy involved will be considered in the *Tripraśnādhikāra*. The process of finding the declination of a planet having either a north or south celestial latitude is rough. Brahmagupta and the modern *Sūrya-siddhānta* follow the same rule. Bhāskara alone attempted to make it somewhat correct. Cf. *Brāhmasphuṭa-siddhānta*, vii, 15 also x, 15; *Sūrya-siddhānta*, ii, 58; Bhāskara's *Grahagaṇita*, *Grahacchāyādhikāra*, 3; Lalla's *Śiṣyadhīvṛddhida*, xi, 12.

Next in calculating the moon's celestial latitude, it is necessary to find the 'sines' of given arcs, hence a table of 'sines' is also necessary which is also given in the *Tripraśnādhikāra*. Pṛthūdaka takes that up here.

30. Thirty increased severally by nine, six and one; twenty-four, fifteen and five, are the tabular differences of 'sines' at intervals of half a sign. For any arc, the 'sine' is the sum of the parts passed over, increased by the proportional part of the tabular difference to be passed over.

The tabular differences of "sines" are

39, 36, 31, 24, 15, 5.

The "sines" are:

39, 75, 106, 130, 145, 150.

Here the radius of the circle is 150; hence the calculated 'sines' are:—

38·82, 75, 106·06, 129·94, 144·89, 150.

Hence Brahmagupta's "sines" are all accurate to the nearest integer. It is further necessary to find the moon's celestial latitude and the angular diameters of the sun and the moon, for which we have the lines in iv, 1-2 which are taken up here.

31. From the longitude of the moon, subtract that of the ascending node; of the resulting arc, the 'sine' multiplied by 9 and divided by 5 gives the celestial latitude of the moon. Multiply the apparent daily motions of the sun and the moon respectively by 11 and 10, and divide respectively by 20 and 247; the resulting minutes are the angular diameters of the sun and the moon.

The first part says that the celestial latitude of the moon

$$= \frac{9 \times 150 \sin \text{(arc between moon and node)}}{5} \text{ min.}$$

$$= \frac{270 \times 150 \sin \text{(arc between moon and node)}}{150} \text{ min.}$$

The maximum celestial latitude of the moon is thus taken at 270′ as in all Indian *Siddhāntas*.

As to the second part, the idea is that all planets move with the same linear speed in their eccentric circles. Hence if the distance of a planet from the earth (in this case) be r, and n be the daily motion, then $r \times n = \text{constant}$.

Thus if a be mean distance and w be the mean daily motion and r and n be these quantities on any given day, we have $rn = aw$.

$$\therefore\; a : r = n : w.$$

Again if D be the diameter of the planet in linear measure, then the angular diameter,

$$= \frac{D}{r} \times 3438'.$$

The mean angular diameter $= \dfrac{D}{a} \times 3438'$

$$\therefore\; \frac{\text{apparent angular diameter}}{\text{mean angular diameter}} = \frac{a}{r} = \frac{n}{w}.$$

$\therefore$ in the case of the sun, the apparent angular diameter

$$= \frac{\text{mean angular diam.} \times n}{w}$$

$$= \frac{11}{20} \times n\,;$$

$$\therefore\; \frac{11}{20} = \frac{\text{mean angular diameter}}{59'\,8''},$$

$$\therefore\; \text{the sun's mean angular diameter} = \frac{650'\,28''}{20} = 32'\,31''.$$

Similarly the moon's mean angular diameter

$$= \frac{790'\,34'' \times 10}{247} = 32' \text{ nearly.}$$

The next stanza that is necessary here, is taken by Pṛthūdaka, from the *Tripraśnādhikāra*; it relates to the finding of the arc when the 'sine' is known.

32. Subtract as many as possible of the tabular differences of the 'sines' from the given 'sine'; multiply the remainder by 900 and divide by the tabular difference that cannot be subtracted; add the resulting minutes to 900′ multiplied by the *number* of tabular differences passed over; the final result will be the arc corresponding to the given 'sine.'

The tabular differences of 'sines' are given at the intervals of 15° or 900′. The rest requires no explanation. All the rules necessary for a computation of a *vyatipāta* or *vaidhṛta* having been discussed, Pṛthūdaka illustrates it as follows:—

Illustration.—Let the time be 786 *Saka* year, 1 synodic month and 10 *tithis*; then *ahargaṇa*=72714, the mean sun as corrected for Kurukṣetra=0 sign 27° 37′ 40″; the sun's apogee = 2 signs 20°, ∴ the sun's mean anomaly = 10 signs 7° 37′ 40″, the sun's equation 1° 45′ 19″ according to Brahmagupta's table: hence the apparent sun at midnight = 0 sign 29° 22′ 59″; the sun's daily motion, 57′ 46″.

The moon's mean longitude = 5 signs 4° 7′ 12″, the moon's apogee = 4 signs 8° 49′ 5″, ∴ the moon's mean anomaly = 0 sign 25° 17′ 7″; hence the moon's equation = 2° 6′ 39″; the apparent moon = 5 signs 2° 0′ 33″; the moon's daily motion = 728′ 39″.

Sun = 0 sign 29° 22′ 59″;

Moon = 5 signs 2° 0′ 33″

Sum = 6 signs 1° 23′ 32″; which is in excess of 6 signs by 1° 23′ 32″ or 83′ 22″.

$$\text{Now, } \frac{83'\ 32''}{\text{Sun's daily motion}+\text{Moon's daily motion}} = \frac{83'\ 32''}{786'\ 25''} \text{ days}$$

represent the time when sun + moon will be 6 signs or 180°.

$$\therefore \text{ correction to sun} = \frac{83'\ 32'' \times 57'\ 46''}{786'\ 25''} = 6'\ 8''.$$

Correction to moon=83′ 32″−6′ 8″= 77′ 24″.

Longitude of the moon's ascending node=4 signs 5° 27′ 46″.

$$\text{Correction to the node} = \frac{83'\ 32'' \times 3'\ 11''}{786'\ 25''} = 20''.$$

Thus at the time when the sum of the longitudes of the sun an the moon is 180°, the longitudes are:—

Of the sun= 0 sign 29° 16′ 51″

Of the moon= 5 signs 0° 43′ 9″

Of the node= 4 signs 5° 28′ 6″.

These are the longitudes at $\frac{83'\ 32''}{786'\ 25''}$ da. or 6 *ghaṭikās* and 22 *pal*ᵃ preceding the midnight. At this time from the sun's longitude =0 sign 29° 16′ 51″, his declination= 11° 28′ 32″=688′ 32″.

Now,

Moon	= 5 signs 0° 43′ 9″;
Moon's node	= 4 signs 5° 28′ 6″;
∴ Moon − node	= 0 sign 25° 15′ 3″;

∴ the moon's celestial latitude= 115′ 13″.

Hence by the rule of the *Khaṇḍakhādyaka*, the moon's declination=688′ 32″+115′ 13″ = 803′ 45″. Hence the declinations are not equal and the time found is not the time for *vyatipāta*. The next step is to subtract the moon's celestial latitude from its declination and to find the moon's longitude for which the declination is equal to the remainder of the subtraction.

Sun's declination= 688′ 32″;

Moon's celestial latitude= 115′ 45″;

Difference= 572′ 47″.

Now 572′ 47″ is the declination for the longitude 24° 3′ 48″ or 5 signs 5° 56′ 12″.

Hence the longitude of the moon at which the moon's declination will be nearest to that of the sun= 5 signs 5° 56′ 12″.

The moon at midnight = 5 signs 2° 0′ 33″.

Difference = 3° 55′ 39″.

The moon's daily motion being 728′ 59″, the corresponding time is $\frac{235'\ 39'' \times 60}{728'\ 59''}$ *ghaṭikās* = 19 *gh.* 44 *palas*, after midnight. The motions of the sun and the node are respectively in these 19 *gh.* 44 *palas*=19′, and 1′ 3″.

Hence at this time, *i.e.*, 19 *gh.* 44 *palas* after midnight, the ngitudes of—

The sun	= 0 sign 29° 41′ 59″,
The moon	= 5 signs 5° 56′ 12″,
and the node	= 4 signs 5° 26′ 43″.
Now the sun's declination =	11° 37′ 33″.
The moon's ,, =	9° 49′ 41″.
,, ,, celestial latitude =	2° 17′ 3″.

Again, sun's declination — moon's celestial latitude $= 9° \, 20' \, 30''$.

Taking $9° \, 20' \, 30''$ to be the moon's declination,

the moon's longitude $=$ 5 signs $6° \, 28' \, 43''$.

The moon's longitude at midnight $=$ 5 signs $2° \; 0' \, 33''$

Difference $= 4° \, 28' \, 10''$

∴ the next approximation to the time of the *vyatipāta* is

$$\frac{4° \, 28' \, 10''}{728' \, 39''} \text{ days, } i.e., \; \frac{4° \, 28' \, 10'' \times 60}{728' \, 39''} \; gh.$$

$= 22$ *gh.* 3 *palas* after midnight.

At that time the sun's longitude	$=$ 0 sign $29° \, 44' \, 13''$.
The longitude of the node	$=$ 4 signs $5° \, 26' \, 36''$.
The longitude of the moon	$=$ 5 signs $6° \, 28' \, 43''$.
Now the sun's declination	$= 11° \, 38' \, 21''$;
the moon's declination	$= 9° \, 20' \, 30''$;
and the moon's celestial latitude	$= 2° \, 19' \, 15''$.

Again, the sun's declination − moon's celestial latitude $= 9° \, 19' \, 6''$;
taking this to be the moon's declination,

the moon's longitude $=$ 5 signs $6° \, 32' \, 27''$.

The moon's longitude at midnight $=$ 5 signs $2° \; 0' \, 33''$,

∴ difference $= 4° \, 31' \, 54''$;

∴ the final approximation to the *vyatipāta* is

$$\frac{4° \, 31' \, 54'' \times 60}{728' \, 39''} \; ghaṭikās \text{ or } 22 \; gh. \; 23 \; palas \text{ after midnight.}$$

At that time the sun's longitude	$=$ 0 sign $29° \, 44' \, 32''$;
the longitude of the moon's node	$=$ 4 signs $5° \, 26' \, 35''$;
the longitude of the moon	$=$ 5 signs $6° \, 32' \, 27''$;
the sun's declination	$= 11° \, 38' \, 28''$;
the moon's declination	$= 9° \, 19' \, 6''$;
the moon's celestial latitude	$= 2° \, 19' \, 27''$;
∴ the moon's true declination	$= 11° \, 38' \, 33''$.

Now the declination of the moon may be practically taken eq to that of the sun. The time in which this equality or *vyatipāta* takes place is 22 *gh.* 23 *palas* after midnight. The length of half the night is 13 *gh.* 51 *palas.* Hence this *vyatipāta* takes place after 8 *gh.* 32 *palas* after the next sunrise.

Now the diameter of the sun $= \frac{11}{20} \times 57' \, 46'' = 31' \, 46''$.

the diameter of the moon $= \frac{10}{247} \times 728' \, 39'' = 29' \, 30''$;

∴ half the sum of the semi-diameter $= 30' 38''$; this is gained by the moon in 2 *gh.* 44 *palas.* Therefore the *vyatipāta* begins at 5 *gh.* 48 *palas* and ends at 11 *gh.* 16 *palas* after the sunrise.

As to the duration of the *vyatipāta*, Bhāskara says—

"यदा विम्बमध्ययोः क्रान्तिसाम्यं तदा पातमध्यम् ।
तदनन्तरं रवेरग्रप्रान्तस्य चन्द्रपृष्ठप्रान्तस्य च यदा क्रान्तिसाम्यं तदा पातान्तः * ।"

i.e., "it is the middle of the *pāta*, when the centres of the sun and the moon have the same declination; the *pāta* ends later on when the foremost part of the sun's disc and the hindmost part of the moon's disc have the same declination." Hence the calculation of the duration of a *pāta* has the same nature as the calculation of an eclipse.

This practically finishes the first chapter of the *Khaṇḍakhādyaka.* Pṛthūdaka gives some additional rules which are detailed below.

(*i*) The calculation of the duration of the sun's passage from one sign of the zodiac to the next. The directions are (*a*) first find the instant when the sun's centre is at the junction of the two signs; (*b*) then multiply by 60, the sun's semi-diameter, and divide by the sun's apparent daily motion; apply the quotient which is in *ghaṭikās* negatively and positively to the instant when the centre of the sun is at the junction of the two signs. Thus are obtained the instants of the sun's transit from one sign of the zodiac to the next. In the same way find the durations of the end of *nakṣatras* or of *tithis.*

(*ii*) The next topic treated by Pṛthūdaka, is to find the Lord of the year. This rule is: add 319 to the *ahargaṇa* as found from the *Khaṇḍakhādyaka*, and divide by 360; multiply the quotient by 3 and increase the product by 3; divide the result by 7; the remainder counting from Sun in the week day order gives the Lord of the year.

Illustration.—Let the *ahargaṇa* be 72675, then $72675 + 319 = 360 \times 202 + 274$; again $202 \times 3 + 3 = 7 \times 87 + 0$. Here the remainder is 0 or 7, hence Saturn is the Lord of the year of which 274 days

* *Grahagaṇita, Pātādhikāra,* 15-16, Comm.

have passed and will continue to be the Lord of the year for 86 days more.

(*iii*) To find the Lord of the month, add 19 to the *Khaṇḍakhādyaka ahargaṇa*, and divide by 30 ; double the quotient and add 2 ; now divide by 7 ; the remainder of this division gives the Lord of the month from the Sun in the week day order.

Illustration :—Let the *ahargaṇa* be 72675 as before ; increased by 19 it becomes 72694. Now $72694=30\times2423+4$ and $2423\times2+2=7\times692+4$. Thus Mercury is the Lord of the month.

(*iv*) The Lord of the day is found from the *ahargaṇa*. The Lord of the hour is thus found: find the number of hours elapsed since sunrise, multiply the number by 5, add 1 to the product, divide by 7. The remainder counted in the week day order, from the Lord of the day, gives the Lord of the hour.

Illustration.—It is proposed to find the Lord of the sixth hour of a Tuesday.

The number of hours elapsed is 5 ; now $5\times5+1=26=7\times3+5$, hence Saturn is the Lord of the sixth hour of a Tuesday.

This finishes the first chapter of the Khaṇḍakhādyaka which relates to the finding of tithis, nakṣatras, etc.

CHAPTER II

On the Mean and True Places of 'Star' Planets.

1. Deduct from the *ahargaṇa* $496-\frac{1}{4}$, and divide by 687 ; the result is the mean Mars in revolutions, etc. ; again divide the *ahargaṇa* by 174259, add the quotient taken as minutes to the revolutions, etc., obtained before.

Thus mean Mars $=\dfrac{(ahargaṇa-496+\frac{1}{4})}{687}$ revols. $+\dfrac{ahargaṇa}{174259}$ min.

Now in a *Mahāyuga*, the number of civil days=1577917800 ;

∴ the number of Mars' revolutions in a *Mahāyuga*

$$\frac{1577917800}{687}\text{ rev.}+\frac{1577917800}{174259\times60\times360}\text{ rev.}$$

$$=\left(2296823+\frac{399}{687}+\frac{292207}{697036}\right)\text{ revolutions}$$

$$=(2296823\cdot580786+\cdot419214)\text{ revolutions}$$

$$=\mathbf{2296824}\text{ revolutions.}$$

Thus according to the *Khaṇḍakhādyaka*, the number of Mars' revolutions in a *Mahāyuga* is taken at **2296824**.

The mean Mars, therefore

$$=\frac{ahargaṇa\times2296824}{1577917800}\text{ revols.}$$

$$=\frac{ahargaṇa}{687-\dfrac{288}{2296824}}\text{ revols.}$$

$$=\left\{\frac{ahargaṇa}{687}+\left(\frac{ahargaṇa}{687-\dfrac{288}{2296824}}-\frac{ahargaṇa}{687}\right)\right\}\text{ revols.}$$

$$=\frac{ahargaṇa}{687}\text{ rev.}+ahargaṇa\times\frac{1}{\dfrac{687\times292207}{288\times2\times2}}\text{ min.}$$

$$=\frac{ahargana}{687}\text{ rev.}+\frac{ahargana}{174259-\frac{152}{1152}}\text{ min.}$$

$$=\frac{ahargana}{687}\text{ rev.}+\frac{ahargana}{174259}\text{ min., rejecting the fraction }\frac{152}{1152}$$

in the denominator.

Again the *ahargaṇa* till the beginning of the *Khaṇḍakhādyaka* epoch $=1375565=687\times2002+191$.

$$\text{Again}\quad\frac{1375565\text{ revols.}}{174259\times60\times360}$$

$$=\frac{\frac{1375565\times687}{174259\times60\times360}}{687}=\frac{\cdot25}{687}\text{ nearly.}$$

∴ 191·25 is the positive additive. Hence the negative *Kṣepa* $=687-191\cdot25=496-\frac{1}{4}$.

The true value of the numerator of the fraction $\frac{\cdot25}{687}$ is ·25105.

Hence the excess of Mars' revolution left out $=\frac{\cdot00105}{687}$ revols.

$=1''\cdot9\,89$, for which Brahmagupta directs the addition of $2''$ to the calculated mean longitude of Mars, Ch. I, st. 7.

Illustrations.—(*i*) Take 1 for the *ahargaṇa*, then the daily motion of Mars $=\frac{1}{687}$ rev. $+\frac{1}{174259}$ min.

$$=31'\ 26''+0''=31'\ 26''.$$

(*ii*) Let the time be *Śaka* year elapsed 786. The *ahargaṇa* is 72675

$$\therefore\text{ the mean Mars}=\frac{(72675-496+\frac{1}{4})}{687}\text{ rev.}+\frac{72675}{174259}\text{ min.}$$

$$=\frac{72179\cdot25}{687}\text{ rev.}+\frac{72675}{174259}\text{ min.}$$

$$=105\text{ rev. }0\text{ sign }23^\circ\ 11'\ 16''+25''$$

$$=0\text{ sign }23^\circ\ 11'\ 41''.$$

To this must be added $2''$ according to stanza 7 of Chapter I.

∴ the mean Mars at the midnight of Laṅkā, at the end of 786 of *Śaka* year $=0$ sign $23^\circ\ 11'\ 43''$.

2. Multiply the *ahargaṇa* by 100, and lessen the product by 2181 and divide it by 8797; the quotient is the *Śīghra* of Mercury in revolutions. Increase the result by the minutes of arc obtained from the *ahargaṇa* divided by 71404.

Here the *Śighra* of Mercury

$$=\frac{ahargana\times100-2181}{8797}\text{ rev.}+\frac{ahargana}{71404}\text{ min.}$$

The number of civil days in a *Mahāyuga* is 1577917800; hence the number of revolutions of Mercury in a *Mahāyuga*

$$=\frac{1577917800\times100}{8797}\text{ rev.}+\frac{1577917800}{71404}\text{ min.}$$

$$=(17936998\cdot9769+1\cdot0231)\text{ rev.,}$$

$$=\mathbf{17937000}\text{ rev.}$$

Thus the number of revolutions of Mercury in a *Mahāyuga*, as accepted in the *Khaṇḍakhādyaka* is **17937000.**

∴ the longitude of the *Śighra* of Mercury

$$=\frac{ahargana\times17937000}{1577917800}\text{ rev.}=\frac{ahargana\times100}{\frac{1577917800}{179370}}\text{ rev.}$$

$$=\frac{ahargana\times100}{8797}\text{ rev.}+\left(\frac{ahargana\times100}{8797-\frac{9}{17937}}-\frac{ahargana\times100}{8797}\right)\text{rev.}$$

$$=\frac{ahargana\times100}{8797}\text{ rev.}+\frac{ahargana}{\frac{8797\times292207}{360000}}\text{ min.}$$

$$=\frac{ahargana\times100}{8797}\text{ rev.}+\frac{ahargana}{71404\frac{2}{25}}\text{ min.}$$

$$=\frac{ahargana\times100}{8797}\text{ rev.}+\frac{ahargana}{71404}\text{ min., rejecting the fraction }\frac{2}{25}$$

in the denominator of the second term. This proves Brahmagupta's rule.

Again the *ahargaṇa* till the beginning of the *Khaṇḍakhādyaka* epoch$=1375565$,

and $137556500 = 8797 \times 15636 + 6608$

$$\text{Also } \frac{1375565}{71404} \text{ min.} = \frac{1375565 \times 8797}{\frac{71404 \times 21600}{8797}} \text{ rev.}$$

$$= \frac{7 \cdot 84584}{8797} \text{ rev.}$$

$\therefore$ the positive additive$= 6608 + 7 \cdot 84584$
$= 6615 \cdot 84584.$

$\therefore$ the negative *kṣepa* $= 2181 \cdot 15416$, this is taken at 2181 and the error is $\frac{\cdot 15416}{8797}$ rev.$= 22'' \cdot 716$ seconds.

In its place Brahmagupta directs the subtraction of $22''$ seconds from the longitude of Mercury, *vide* Ch. I, 7.

Illustration :—(*a*) Let the *ahargaṇa* be 72675.

$$\text{The mean Mercury} = \frac{7267500 - 2181}{8797} \text{ rev.} + \frac{72675}{71404} \text{ min.}$$

$$= 825 \text{ rev. } 1 \text{ sign } 23° \; 9' \; 33'' + 1' \; 1''$$
$$= 1 \text{ sign } 23° \; 10' \; 34''.$$

From this $22''$ have to be subtracted.

Thus the mean Mercury $=$ 1 sign $23° \; 10' \; 12''$ at midnight at Laṅkā.

(*b*) To find the daily motion of Mercury.

$$\text{It is} = \frac{21600 \times 100}{8797} \text{ min.} + \frac{1}{71404} \text{ min.}$$

$$= 4° \; 5' \; 32''.$$

3. Deduct from the *ahargaṇa* $2113 - \frac{1}{3}$, and divide by 4332, the quotient in revolutions, etc., is the mean Jupiter, when diminished by the number of degrees obtained from the *ahargaṇa* divided by 162621.

Here the mean longitude of Jupiter

$$= \frac{ahargaṇa - 2113 + \frac{1}{3}}{4332} \text{ rev.} - \frac{ahargaṇa}{162621} \text{ degrees.}$$

Hence the number of Jupiter's revolutions in a *Mahāyuga*

$$= \frac{1577917800}{4332} \text{ rev.} - \frac{1577917800}{162621} \text{ degrees.}$$

$$= 364246 \cdot 95290 - 26 \cdot 95284 \text{ rev.}$$

$$= 364220 \cdot 00006 \text{ rev.} = 364220.$$

Thus according to the *Khaṇḍakhādyaka*, the number of Jupiter's sidereal revolutions is accepted at **364220**.

Hence the mean longitude of Jupiter

$$= \frac{ahargaṇa \times 364220}{1577917800} \text{ revols.}$$

$$= \frac{ahargaṇa}{4332 + \frac{116760}{364220}} \text{ revols.}$$

$$= \frac{ahargaṇa}{4332} \text{ rev.} - \left\{ \frac{ahargaṇa}{4332} - \frac{ahargaṇa}{4332 + \frac{116760}{364224}} \right\} \text{ rev.}$$

$$= \frac{ahargaṇa}{4332} \text{ rev.} - \frac{ahargaṇa}{162621 - \frac{285}{1946}} \text{ degrees.}$$

$$= \frac{ahargaṇa}{4332} \text{ rev.} - \frac{ahargaṇa}{162621} \text{ degrees,}$$ rejecting the small fraction in the denominator.

Again the *ahargaṇa* till the beginning of the *Khaṇḍakhādyaka* epoch$= 1375565$,

and $1375565 = 4332 \times 317 + 2321.$

$$\text{Again } \frac{1375565}{162621} \text{ degrees} = \frac{1375565 \times 4332}{\frac{162621 \times 360}{4332}} \text{ rev.}$$

$$= \frac{101 \cdot 78656}{4332} \text{ rev.}$$

Now $2321 - 101 \cdot 78656$ is the positive additive.

$\therefore$ $4332 - 2321 + 101 \cdot 78656$ or

$2112 \cdot 78656$ is the negative *kṣepa* : in its place Brahmagupta takes $2112 \cdot 8$ for the negative *kṣepa*.

Thus there remains a residue of

$\frac{\cdot 01344}{4332}$ of a revolution $=4''$. Hence $4''$ are to be added to the mean longitude of Jupiter as determined by the rule.

Illustration :—(*a*) Let the *ahargaṇa* be 72675.

Then the mean longitude of Jupiter

$$=\frac{(72675\frac{1}{5}-2113)}{4332}\text{ rev.}-\frac{ahargaṇa}{162621}\text{ degrees.}$$

$= 16$ rev. 3 signs $13^\circ\ 53'\ 41''-0^\circ\ 26'\ 49''$

$=3$ signs $13^\circ\ 26'\ 52''$, omitting the entire revolutions.

To this mean longitude by adding $4''$, we get for the midnight at Laṅkā, the mean longitude of Jupiter,

$=3$ signs $13^\circ\ 26'\ 56''$.

(*b*) To find the mean daily motion of Jupiter.

It is $=\frac{21600'}{4332}-\frac{60\times 60}{162621}$ seconds,

$=4'\ 59''-0''=4'\ 59''$.

4. Deduct from the *ahargaṇa* $37\frac{1}{4}$ and multiply by 10 and divide by 2247, the quotient in revolutions etc., is the mean Venus-*Śīghra* (heliocentric Venus) when increased by the number of degrees from the *ahargaṇa* lessened by 712 and divided by 77043.

The mean longitude of the *Śīghra* of Venus is thus

$$=\frac{(ahargaṇa-37\frac{1}{4})10}{2247}\text{ rev.}+\frac{ahargaṇa-712}{77043}\text{ degrees.}$$

Hence the number of the sidereal revolutions of Venus in a

Mahāyuga $=\frac{1577917800\times 10}{2247}$ rev. $+\frac{1577917800}{77043\times 360}$ rev.

$=7022331\cdot 10814$ rev. $+56\cdot 89167$ rev.

$=7022387\cdot 99981$

$=$ **7022388** nearly.

Thus according to the *Khaṇḍakhādyaka*, the number of sidereal revolutions of Venus is taken at **7022388**.

The longitude of the *Śīghra* of Venus

$$=\frac{ahargaṇa\times 7022388}{1577917800}\text{ revolutions,}$$

$$=\frac{ahargaṇa\times 10}{15779178000}\text{ rev.}=\frac{ahargaṇa\times 10}{2247-\frac{127836}{7022388}}\text{ rev.}$$

$$=\frac{ahargaṇa\times 10}{2247}\text{ rev.}+ahargaṇa\times 10\left(\frac{1}{2247-\frac{127836}{7022388}}-\frac{1}{2247}\right)\text{rev.}$$

$$=\frac{ahargaṇa\times 10}{2247}\text{ rev.}+\frac{ahargaṇa}{77043-\frac{32013}{127836}}\text{ degrees.}$$

$=\frac{ahargaṇa\times 10}{2247}$ rev. $+\frac{ahargaṇa}{77043}$ degrees, neglecting the small fraction in the denominator which $=\cdot 2504$. This proves the rule of Brahmagupta.

Again the *ahargaṇa* till the beginning of the *Khaṇḍakhādyaka* epoch $=1375565$.

Now $13755650=2247\times 6121+1763$.

Again $\frac{13755650}{77043}$ degrees $=\frac{\frac{13755650\times 2247}{77043\times 360}}{2247}$ rev.

$=\frac{111\cdot 44244}{2247}$.

The positive additive $=1763+111\cdot 44244$

$=1874\cdot 44244$

$\therefore$ the negative additive $=2247-1874\cdot 44244$

$=372\cdot 55756$.

Again Brahmagupta's *kṣepa* quantity

$$=\frac{37\frac{1}{4}\times 10}{2247}\text{ rev.}+\frac{712}{77043}\text{ degrees.}$$

$$=\frac{372\cdot 5}{2247}\text{ rev.}+\frac{\cdot 057586}{2247}\text{ rev.}$$

$$=\frac{372\cdot 557586}{2247}\text{ revolutions.}$$

As calculated, the *kṣepa* $= \frac{372 \cdot 55756}{2247}$ rev.

$\therefore$ difference $= \frac{\cdot 000026}{2247}$ rev.

$= \cdot 014''$ secs. which is neglected.

Illustration :—(*a*) Let the *ahargaṇa* be 72675 as before.

Then the mean Venus's *Śighra* $= \frac{(72675 - 37\frac{3}{4})10}{2247}$ rev. $+ \frac{72675 - 712}{77043}$ degrees,

$= 323$ rev. 3 signs $5° \; 34' \; 3'' + 0° \; 56' \; 2''$,

$= 3$ signs $6° \; 30' \; 5''$.

(*b*) To find the mean daily motion of the *Śighra* of Venus.

It is $= \frac{10 \times 1296000}{2247}$ secs. $= 1° \; 36' \; 8'' + 0''$.

$= 1° \; 36' \; 8''$.

5. Deduct from the *ahargaṇa* $2491\frac{1}{2}$ and divide by 10766; the quotient in revolutions, etc., is the mean Saturn when lessened by the minutes of arc obtained from the *ahargaṇa* divided by 80450.

The stanza says that the mean longitude of Saturn

$$= \frac{ahargana - 2491\frac{1}{2}}{10766} \text{ rev.} - \frac{ahargana}{80450} \text{ min.}$$

Hence the number of sidereal revolutions of Saturn in a *Mahāyuga*

$$= \frac{1577917800}{10766} \text{ rev.} - \frac{1577917800}{80450 \times 360 \times 60} \text{ rev.}$$

$= 146564 \cdot 908044$ rev. $- \cdot 908039$ rev.

$= 146564 \cdot 000005$ rev. $=$ **146564** revolutions.

$\therefore$ the mean longitude of Saturn

$$= \frac{ahargana \times 146564}{1577917800} \text{ revolutions,}$$

$$= \frac{ahargana}{10766 + \frac{9776}{146564}} \text{ revolutions,}$$

$$= \frac{ahargana}{10766} \text{rev.} + ahargana \left\{ \frac{1}{10766 + \frac{9776}{146564}} - \frac{1}{10766} \right\} \text{rev.}$$

$$= \frac{ahargana}{10766} \text{ rev.} - \frac{ahargana}{80450 - \cdot 41504} \text{ min.}$$

$$= \frac{ahargana}{10766} \text{ rev.} - \frac{ahargana}{80450} \text{ min.,}$$ neglecting the fraction $\cdot 415$ in the denominator of the second term. This proves Brahmagupta's rule.

Again the *ahargaṇa* till the beginning of the *Khaṇḍakhādyaka* epoch $= 1375565$.

$1375565 = 10766 \times 127 + 8283$.

Again $\frac{1375565 \times 10766}{80450 \times 21600} = 8 \cdot 52229$.

Hence the positive additive $= 8274 \cdot 47771$; the negative *kṣepa* $= 10766 - 8274 \cdot 47771 = 2491 \cdot 52229$; in its place Brahmagupta takes $2491 \cdot 5$. Thus a residue of $\frac{\cdot 0229 \times 1296000}{10766}$ seconds, *i.e.*, $\frac{229 \times 129 \cdot 6}{10766}$ secs. $= 2'' \cdot 76$ seconds or $3''$, must be subtracted from the mean longitude of Saturn as found from the rule, *vide* Ch. i, 7.

Illustration :—(*a*) Let the *ahargaṇa* be 72675 as before. The mean longitude of Saturn at midnight at Laṅkā

$$= \frac{72675 - 2491 \cdot 5}{10766} \text{ rev.} - \frac{72675}{80450} \text{ min.}$$

$= 6$ signs $6° \; 50' \; 17'' - 54''$; subtracting $3''$ also we get the longitude of Saturn $= 6$ signs $6° \; 49' \; 20''$.

(*b*) To find the mean daily motion of Saturn.

It is $= \frac{21600}{10766}$ min. $= 2'$ min.

6 (1st half). Of planets beginning with Mars, the degrees of longitude of the apogees are respectively 11, 22, 16, 8 and 24, each multiplied by 10.

The longitude of the apogee of Mars = 110° = 3 signs 20°.
" " " Mercury = 220° = 7 signs 20°.
" " " Jupiter = 160° = 5 signs 10°.
" " " Venus = 80° = 2 signs 20°.
" " " Saturn = 240° = 8 signs 0°.

Cf. the *Sūrya-siddhānta* of the *Pañcasiddhāntikā*, xvii, 2.

6 (2nd half) and 7. Of Venus the equation of apsis is like (*i.e.*, the same as) that of the sun, of the son of the moon (*i.e.*, Mercury) is the same doubled, of Mars it is five times, of Jupiter the same increased by $\frac{1}{7}$th and doubled, of Saturn it is the same increased by $\frac{1}{14}$th and quadrupled.

This stanza says that the equations of apsis of the "star planets" are to be obtained from those of the sun in the following way:—

Venus's equation = Sun's equation,
Mercury's " = Sun's equation × 2,
Mars's " = Sun's equation × 5,
Jupiter's " = Sun's equation × $(1+\frac{1}{7}) \times 2$,
Saturn's " = Sun's equation × $(1+\frac{1}{14}) \times 4$.

The dimension of the sun's epicycle of apsis being 14°, it follows that the dimensions of the epicycles of apsis are, for Venus 14°, for Mercury 28°, for Mars 70°; for Jupiter 32°, for Saturn 60°.

These are in agreement with those of the *Sūrya-siddhānta* of the *Pañcasiddhāntikā*, and those of the *Paulisa tantra*, as quoted by Āmaraja. *Cf.* the editor's paper *Āryabhaṭa*, already referred to. It is further clear that the corrections to the mean motions for the apparent motions of these planets are to be obtained from the corresponding corrections to the sun's mean motion by using the same multipliers. We shall consider the different dimensions of the epicycles in the appendix.

8-9. Mars, by the degrees of *Śīghra* anomaly (*i.e.*, anomaly of conjunction) of 28, getting at the corresponding equation of 11° rises (heliacally) in the east; by the next 32° gets 12° more of the equation; by the next 30°, 10° more; by next 31°, 7° more; by next 14°, half a degree; these are positive; by the next 13°, negative 3°; by the next 16°, negative 12°; after this he is retrograde; by the next 9°, negative 13°, by the next 7°, negative 12½°. After this the parts of the equations occur in the reverse order.

The above two stanzas give the parts of the equations of conjunction of Mars thus:—

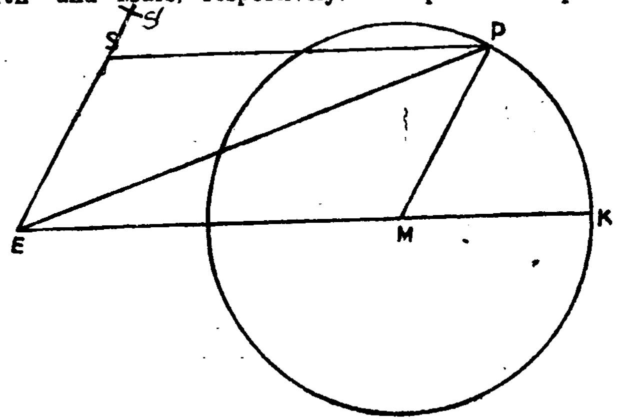
Increase of the anomaly of conjunction...	28°	32°	30°	31°	14°	13°	16°	9°	7°
Increase of the equation of conjunction...	11°	12°	10°	7°	0°30′	3°	12°	13°	12°30′
Nature of the parts	Positive					Negative.			

This means the following table of *Śīghra* equations for Mars:—

Degrees of anomaly of conjunction	Equation of conjunction	Phenomena
0°	0°	Motion direct.
28°	+11°	Rises in the east.
60°	+23°	
90°	+33°	
121°	+40°	
135°	+40° 30′	
148°	+37° 30′	
164°	+25° 30′	Retrograde motion begins.
173°	+12° 30′	
180°	0° 0′	
187°	−12° 30′	
196°	−25° 30′	Direct motion begins.
212°	−37° 30′	
225°	−40° 30′	
239°	−40° 0′	
270°	−33° 0′	
300°	−23° 0′	
332°	−11° 0′	Sets in the west.
360°	0° 0′	

These *Śighra* equations are calculated on the supposition that Mars's epicycle of conjunction has a periphery of 234°, when the circumference of the concentric is 360°. The radii of these circles may be taken to be 234 and 360 units respectively.

In the figure given below, let S, E, P, be the positions of the sun, the earth and Mars, respectively. Complete the parallelogram



$SEMP$; with M as centre and MP for the radius describe a circle. This circle is the epicycle of conjunction of Mars. Produce EM to cut this circle at K. The $\angle PMK = \angle S'SP$,* the angle gained by the earth over Mars since the preceding conjunction. The $\angle PMK$ is called the *Śighra* anomaly or anomaly of conjunction. We take $EM=360$, and $MP=234$. The $\angle PEM$, which is equal to $\angle EPS$, the annual parallax of Mars, is called the *Śighra* equation. The $\angle MPE$ is equal to the $\angle SEP$, the elongation. The $\angle PMK$ is given, and PM and ME are also given. Hence in the triangle MPE, we have

$$\tan \tfrac{1}{2}(P-E) = \frac{EM-MP}{EM+MP} \tan \tfrac{1}{2} PMK,$$

$$= \frac{126}{594} \tan \tfrac{1}{2} PMK.$$

$$\therefore\ L \tan \tfrac{1}{2}(P-E) = \log\left(\frac{126}{594}\right) + L \tan \tfrac{1}{2} PMK. \quad \ldots\ldots\ldots\ldots\ldots (1)$$

We have also

$$\tfrac{1}{2}(P+E) = \tfrac{1}{2} \angle PMK \quad \ldots\ldots \quad (2)$$

Now $\log\left(\frac{126}{594}\right) = \bar{1}\cdot 3265841.$

* The point S' is on ES produced.

The values of the $\angle PMK$ and the $\angle PEM$ and Brahmagupta's values for the latter may be presented in a comparative view:—

$\angle PMK$ =	28°	60°	90°	121°	135°	148°	164°	173°
$\angle PEM$ =	10°58′	23°1′	33°1′	39°56′	40°23′	37°31′	25°32′	12°35′
Brahmagupta's value of $\angle PEM$	11°	23°	33°	40°	40°30′	37°30′	25°30′	12°30′

It will be seen that Brahmagupta gives the values of the equation within $\frac{1}{4}$th of a degree. It seems inexplicable why such discrepancies should remain in Brahmagupta's calculations. It is probable that he wanted to state his equations to the nearest half a degree.

Again he says that Mars's retrograde motion begins when the *Śighra* anomaly is equal to 164°. This requires examination. The longitude l, of P is given by

$l = nt - \theta + E$, where n represents the sun's mean daily motion, t, the number of days elapsed since S was at the origin of celestial co-ordinates, θ is the angle PMK and E stands for the angle PEM. It is evident that nt=longitude of S.

$$\therefore\ \frac{dl}{dt} = n - \frac{d\theta}{dt}\left(\frac{r^2+rp\cos\theta}{r^2+p^2+2rp\cos\theta}\right),$$

$$E = \tan^{-1}\frac{p\sin\theta}{r+p\cos\theta},$$ where $p=MP$ and $r=EM$, and are regarded as constant.

Again $\theta = (n-n')(t+\alpha)$ when n' is the mean daily motion of Mars and α is a constant.

$$\therefore\ \frac{d\theta}{dt} = n-n';$$

$$\text{now } \frac{dl}{dt} = \frac{np^2+n'r^2+pr(n-n')\cos\theta}{r^2+p^2+2pr\cos\theta}.$$

Hence for the stationary point marking the beginning of the retrograde motion, we must have

$$\cos\theta = -\frac{np^2+n'r^2}{pr(n+n')}.$$

Now $n = 59' \, 8''$ and $n' = 31' \, 26''$,

or $n = 3548''$ and $n' = 1886''$,

$n + n' = 5434''$, $p = 234$ and $r = 360$;

$\therefore \cos\theta = -\cdot 959089$,

$\therefore \theta = 163° \, 27'$.

Brahmagupta states its value to be 164°. *Cf.* The modern *Sūrya-siddhānta*, ii, 53. In the *Brāhmasphuṭa-siddhānta* the value given is 163. This last value is copied by Lalla in his *Śiṣyadhībriddhida*, ii, 47.

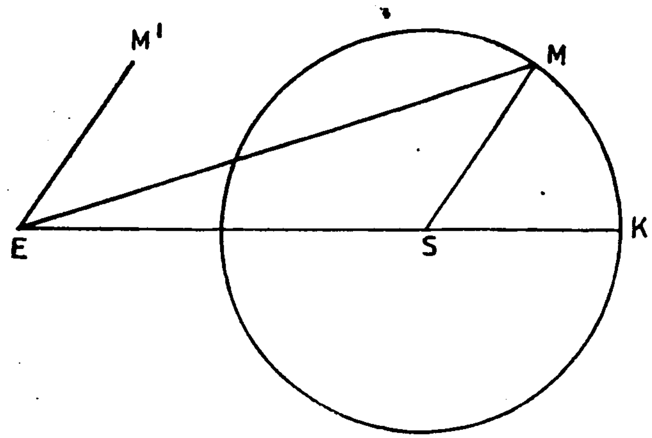
Again the elongation for the heliacal rising of Mars is indicated to be 17°. This is in agreement with *Āryabhaṭīya*, *Gola*, 4, and *Brāhmasphuṭa-siddhānta*, ii, 51.

10-11. The son of the moon (*i.e.*, Mercury), by 51° of *Śīghra* anomaly, getting at the *Śīghra* equation of 13° rises in the west; by 38° more of the *Śīghra* anomaly, 7° more of the equation ; by 31° more of the anomaly 1½ degrees; these parts are positive ; the parts then become negative ; from this point by 26° more of the anomaly, 5°; he then becomes retrograde; by 9° more of anomaly, 3½ degrees; he then sets in the west; then by 25° more of anomaly 13° more of the equation; then in the reverse order east ward.

These stanzas mean the following table for Mercury:—

Increase of *Śīghra* anomaly	Increase of the *Śīghra* equation	*Śīghra* anomaly	*Śīghra* equation	Phenomena
		0°	0°	Motion direct.
51°	+13°	51°	+13°	Rises in the west.
38°	+7°	89°	+20°	
31°	+1° 30′	120°	+21° 30′	
26°	−5°	146°	+16° 30′	Retrograde motion begins.
9°	−3° 30′	155°	+13° 0′	Sets in the west.
25°	−13°	180°	0° 0′	
25°	−13°	205°	−13° 0′	Rises in the east.
9°	−3° 30′	214°	−16° 30′	Direct motion begins.
26°	−5°	240°	−21° 30′	
31°	+1° 30′	271°	−20° 0′	
38°	+7°	309°	−13° 0′	Sets in the east.
51°	+13°	360°	0° 0′	

We now proceed to examine the equations.



Let E, S, M, be respectively the positions of the earth, sun and Mercury. With S as the centre with radius SM describe a circle which is Mercury's orbit or the epicycle.

As in the case of Mars, if ES be taken $=360°$, then SM is here $=132°$ as appears from the mean of the results from the first three figures of the equations. If on this assumption, the equations are worked out, for the values of the angle MSK, *viz.*,

51°, 89°, 120°, 146°, 155°, they become
13° 3′, 20° 1′, 21° 15′, 16° 25′, 13° 4′ ;

while the values given by Brahmagupta are

13°, 20°, 21° 30′, 16° 30′, 13° respectively.

Evidently 21° 30′ in place of 21° 15′ is an error of calculation. If from 21° 30′, the distance SM be calculated, taking $ES = 360$, then SM becomes 133·5 nearly.

From E, draw EM' parallel to SM ; then exactly as in the case of Mars,

$l = nt - \theta + E$, where l is the geocentric longitude of M ;
nt the longitude of M' ; θ the $\angle MSK$,

$$\frac{dl}{dt} = \frac{np^2 + n'r^2 + pr\,(n+n')\cos\theta}{r^2 + 2pr\cos\theta + p^2}$$; where n, n' are the respective angular velocities of M' and S about E, $p = SM$, $r = ES$.

The value of θ for the stationary point is given by

$$\cos\theta = -\frac{np^2 + n'r^2}{pr(n+n')} ;$$

where $n = 4° \, 5' \, 32'' = 14732''$,
and $n' = 59' \, 8'' = 3548''$;

$$\cos\theta = -\frac{716511168}{868685600} = -\cdot8248299,$$

∴ $\theta = 145°34'$ which according to Brahmagupta is 146°.

The elongation for Mercury's heliacal rising is indicated to be 18°, for it is the $\angle MES$ in the figure. This agrees with Āryabhaṭa. Cf. *Āryabhaṭīya, Gola*, 4. According to the *Brāhmasphuṭa-siddhānta* this angle is 14°.

12-13. Of Jupiter getting at the equation of conjunction of $2\frac{1}{3}$ degrees by the anomaly of 14° and rising in the east, the parts of the equation are by 40° more of anomaly, 6° more of the equation; by 36° more of anomaly, 3° more of the equation; by 18° more of the anomaly, 10′ more of the equation; these are the positive parts of the equation and from this are stated the negative parts; by 22° more of the anomaly, negative 1° 30′ more; he then becomes retrograde; by 14° more of anomaly negative 2° more; by the next 20° of anomaly, negative 4° more; by the next 16° of anomaly negative 4° more of 4° more; by the next 16° of anomaly negative 4° more of the equation; then the parts of the anomaly and of the equation occur in the reverse order.

Here we are given the following table for the *Śighra* equations of Jupiter:—

Increase of the *Śighra* anomaly	Increase of the *Śighra* equation	*Śighra* anomaly	*Śighra* equation	Phenomena
		0°	0°	Motion direct.
14°	+2°20′	14°	+ 2° 20′	Rises in the east.
40°	+6° 0′	54°	+ 8° 20′	
36°	+3° 0′	90°	+11° 20′	
18°	+0°10′	108°	+11° 30′	
22°	−1°30′	130°	+10° 0′	Retrograde motion begins.
14°	−2° 0′	144°	+8° 0′	
20°	−4° 0′	164°	+4° 0′	
16°	−4° 0′	180°	0° 0′	
16°	−4° 0′	196°	−4° 0′	
20°	−4° 0′	216°	−8° 0′	
14°	−2° 0′	230°	−10° 0′	Direct motion begins.
22°	−1°30′	252°	−11° 30′	
18°	+0°10′	270°	−11° 20′	
36°	+3° 0′	306°	−8° 20′	
40°	+6° 0′	346°	−2° 20′	Sets in the west.
14°	+2°20′	360°	0° 0′	

Jupiter being a superior planet, the figure for its epicycle will be [illegible] that for Mars. A preliminary examination of the table [illegible] that in the case of Jupiter $MP=72$, when $EM=360$. The [illegible] values of the *Śighra* equation for the values of the *Śighra* [illegible]

[illegible] 14°, 54°, 90°, 108°, 130°, 144°, 164°,

[illegible] following:—

[illegible] 2°2′, 8° 14′, 11° 18′, 11° 28′, 9° 58′, 7° 59′, 8° 54′,

[illegible] they are given by Brahmagupta as:—

2° 20′, 8° 20′, 11° 20′, 11° 30′, 10°, 8°, 4°.

[illegible] a closer agreement could be obtained by taking [illegible]

[illegible] the value of the *Śighra* anomaly for the stationary point [illegible] is given by:—

$$[illegible] = -\frac{np^2 + n' r^2}{pr(n+n')}, \quad \text{where } p=72,\ r=360,\ n=59'\ 8'',\ n'$$

$$[illegible] = -\frac{57143282}{99714240} = -\cdot573070$$

[illegible] = 124° 58′, which may be taken at 125°. This agrees with the *Brāhmasphuṭa-siddhānta*, ii, 48. But the *Khaṇḍakhādyaka* indicates it to be 130°; this latter result is in agreement with the *Sūrya-siddhānta*, ii, 53.

Again the elongation for Jupiter's heliacal rising is indicated to be 11° 40′. This is not in agreement with the *Āryabhaṭīya*.

14-15. The son of Vṛgu (*i.e.*, Venus) getting at the equation of 10° by 24° of *Śighra* anomaly rises in the west; by 39° more of anomaly, gets 16° more of the equation; by 33° more of anomaly, 12° more of the equation; by 27° more of the anomaly, 7° more of the equation; then by 18° more, $1°\frac{1}{4}$; these parts are positive; the next ones are negative; by 13° more of the *Śighra* anomaly, negative $4°\frac{1}{4}$; by 11° more, negative 10° more; he then becomes retrograde; by 12° more, negative 24° more; by 3° more, negative 8° more; from this the parts occur in the reverse order as before.

These stanzas give the table of *Śighra* equations of Ve[...] different values of the *Śighra* anomaly:—

Increase of *Śighra* anomaly	Increase of *Śighra* equation	*Śighra* anomaly	*Śighra* equation	Phenomena
0°	0°	0°	0°	Motion direct. Rises in the west.
24°	+10°	24°	+10°	
39°	+16°	63°	+26°	
33°	+12°	96°	+38°	
27°	+7°	123°	+45°	
18°	+1°15′	141°	+46°15′	
13°	−4°15′	154°	+42°	
11°	−10°	165°	+32°	Retrograde motion begins.
12°	−24°	177°	+ 8°	Sets in the west.
3°	−8°	180°	0°	
3°	−8°	183°	− 8°	Rises in the east.
12°	−24°	195°	−32°	Direct motion begins.
11°	−10°	206°	−42°	
13°	−4°15′	219°	−46°15′	
18°	+1°15	237°	−45°	
27°	+7°	264°	−38°	
33°	+12°	297°	−26°	
39°	+16°	336°	−10°	
24°	+10°	360°	0°	Sets in the east.

This table readily yields the result that *SM* (in the figure for Mercury) is taken at 260, when *SE* is 360. The calculated values of the *Śighra* equations for the values of the *Śighra* anomaly, viz.,

24°, 63°, 96°, 123°, 141°, 154°, 165°, 177°,

are 10° 2′, 25° 51′, 37° 51′, 44° 57′ 46° 1′, 42° 4′, 31° 44′, 7° 48′;

while Brahmagupta's figures are

10°, 26°, 38°, 45°, 46° 15′, 42°, 32°, 8°.

Again the anomaly of conjunction or the *Śighra* anomaly for the stationary point of Venus is given by—

$$\cos\theta = -\frac{np^2+n'r^2}{pr\,(n+n')}, \text{ where } p=260,\ r=360,\ n'=96'\,8'',\ n=59'\,8''.$$

$$\therefore\ \theta = 167°\ 2'.$$

Thus the *Śighra* anomaly works out to be 167° nearly, but Brahmagupta here indicates its value to be 165°. This is in agreement with his *Brāhmasphuṭa siddhānta*, ii, 48; the modern *Sūrya-Siddhānta* gives its value to be 163°, Ch. ii, 54.

In case of an inferior planet, the *Śighra* equation itself represents the elongation. In the case of Venus the elongation for heliacal rising and setting is indicated to be 10° in the west, and in the east the same is indicated to be 8°. The mean of the two values is in agreement with the *Āryabhaṭīya, Gola,* 4.

16-17. Saturn, by 20° of the *Śighra* anomaly, getting at the equation 2°, rises in the east; then by 36° more, gets 3° more of the equation; by the next 20°, 1° more of the equation; by 20° more, 20′ more of the equation: the parts of the equations are positive up to this; henceforth they become negative; by 20° more of the anomaly, negative 20′ of the equation; he then becomes retrograde; by 17° more of the anomaly, negative 1° more of the equation; by 22° more, 2° more; by 25° more 3° more; then parts come in the reverse order.

The above two stanzas give us the table of Saturn's *Śighra* equations; it is as follows:—

Increase of *Śighra* anomaly	Increase of *Śighra* equation	*Śighra* anomaly	*Śighra* equation	Phenomena
0°	0°	0°	0°	Motion direct. Rises in the east.
20°	+2°	20°	+2°	
36°	+3°	56°	+5°	
20°	+1°	76°	+6°	
20°	+0°20′	96°	+6°20′	
20°	−0°20′	116°	+6°	Motion retrograde begins.
17°	−1°	133°	+5°	
22°	−2°	155°	+3°	
25°	−3°	180°	0°	
25°	−3°	205°	−3°	
22°	−2°	227°	−5°	
17°	−1°	244°	−6°	Motion direct begins.
20°	−0°20′	264°	−6°20′	
20°	+0°20′	284°	−6°	
20°	+1°	304°	−5°	
36°	+3°	340°	−2°	
20°	+2°	360°	0°	Sets in the west.

We readily infer that *MP* (in the figure for Mars) is taken to be 40, when *EM*=360. The calculated values of the *Śighra* equations for the *Śighra* anomalies, of—

20°, 56°, 76°, 96°, 116°, 133°, 155°,

are respectively,

1° 58′, 4° 57′, 5° 59′, 6° 23′, 6°, 5° 2′, 2° 59′;

as given by Brahmagupta, they are,

2°, 5°, 6°, 6° 20′, 6°, 5°, 3°.

Brahmagupta's values are to be considered fairly accurate as he used a very rough trigonometrical table.

Again the *Śīghra* anomaly for the stationary point of ~~Mars~~ Saturn i given by

$$\cos\theta = -\frac{np^2+n'r^2}{pr\,(n+n')}\ ;\ \text{where } n=59'\,8'',\ n'=2',\ p=40,\ r=360.$$

$\therefore\ \theta = 113°\ 42.$

Hence according to this calculation Saturn would be at th stationary point at about 114° of the *Śīghra* anomaly ; in its plac the angle is here suggested to be 116°. According to *Brāhma sphuṭa-siddhānta* the angle is 113°, while according to modern *Sūrya siddhānta* it is 115°.

Again the elongation for the heliacal rising of Saturn is indicate to be 18° ; according to Āryabhaṭa it is 15°.

18. To the mean planet apply half of the *Śīghr* equation; from the planet thus corrected, calculate th equation of apsis, and apply half of it to the corrected mea planet. From the mean planet as corrected for the secon time calculate the equation of apsis and apply the who of it to the mean planet with which the calculation bega From the planet, thus corrected, calculate the *Śīghr* equation and apply the whole of it to this last correcte planet, the final result is the apparent geocentri longitude of the planet. The *Śīghra* diminished by the mean becomes the *Śīghra* anomaly.

Here are indicated four distinct operations to find the geocentri longitude of a 'star-planet.' In the third operation to the me planet at the starting is applied an equation of apsis which does n belong to it. This is not intelligible. In any case the first tw operations are not intelligible. The natural steps must be (1) the case of a superior planet to find the heliocentric longitud and in the case of an inferior planet to get at the centre o the circular orbit and (2) in the case of a superior planet to apply t annual parallax to the heliocentric longitude and in the case of a inferior planet to apply the elongation to the centre of the circular orbit. It is difficult to see the necessity for such a complex rule. This very rule occurs in the *Āryabhaṭīya*, *Kālakriyā*, 23 ; *Brāhma-sphuṭa-siddhānta* ii, 40, and also in the *Surya-siddhanta* ii, 44. The concluding part of the stanza defines the term *Śīghra* anomaly. We illustrate the rule from Pṛthūdaka's example.

Illustration.—To find the geocentric longitude of Mars at the *Śaka* year 785 and 12 synodic months. The *ahargaṇa* is 72675. The mean sun as corrected for the midnight at *Kurukṣetra* = 11 signs 19° 11′ 22″. The mean Mars as corrected for midnight at Kurukṣetra = 0 sign 23° 10′ 56″. But in place of these longitudes, Pṛthūdaka takes the mean sun = 11 signs 19° 11′ 19″ and the mean Mars = 0 signs 23° 10′ 50″. Here the mean sun is the *Śīghra* of Mars ; hence the *Śīghra* anomaly,

= Mean sun − mean Mars.
= 11 signs 19° 11′ 19″ − 0 signs 23° 10′ 50″,
= 10 signs 26° 0′ 29″ = 326° 0′ 29″.

Hence by Brahmagupta's table of *Śīghra* equations for Mars, we get the corresponding

Śīghra equation = − 13° 14′ 50″ ;
this halved = − 6° 37′ 25″.
The mean Mars = 0 signs 23° 10′ 50″ ;

∴ the mean Mars as corrected by the 1st operation = 0 signs 16° 33′ 25″.

Longitude of the apogee of Mars = 3 signs 20° 0′ 0″
The mean anomaly = 8 signs 26° 33′ 25″

∴ the sun's corresponding equation from Brahmagupta's table f the sun = + 132′ 51″

∴ Mar's equation of apsis = 5 × sun's equation
= + 664′ 15″

This halved = + 332′ 8″
= + 5° 32′ 8″

The mean Mars as corrected by the 1st operation
= 0 sign 16° 33′ 25.

19″ ∴ the mean Mars as corrected by 2nd operation
= 0 signs 22° 5′ 33″

Now longitude of the apogee of Mars	= 3 signs 20° 0′ 0″	
∴ the mean anomaly	= 9 signs 2° 5′ 33″	

∴ the sun's corresponding equation from Brahmagupta's table

	= 133′ 18″;	
∴ Mars' equation of apsis	= 11° 6′ 30″.	
The mean Mars before 1st operation	= 0 sign 23° 10′ 50″,	
∴ the heliocentric longitude of Mars	= 1 sign 4° 17′ 20″.	
Again the *Śighra* of Mars	= 11 signs 19° 11′ 19″,	
∴ the *Śighra* anomaly	= 10 signs 14° 53′ 59″	

∴ the *Śighra* equation by Brahmagupta's table

= − 17° 24′ 45″.

Now Mars' heliocentric longitude	= 1 sign 4° 17′ 20″,	
∴ Mars' geocentric longitude	= 0 sign 16° 52′ 35″.	

This illustrates the method of the *Khaṇḍakhādyaka*, for finding the geocentric longitudes of Mercury, Venus, Mars, Jupiter, and Saturn. The equations of apsis are given at the interval of 15° ; the results obtained by taking proportional parts are rough. As to the table for *Śighra* equations they are not very accurate and 'proportional parts' would never lead to mathematically correct results.*

19. In the same way are to be found the apparent daily motions of these planets: the daily motion of the *Śighra* diminished by the apparent daily motion in the third operation becomes the divisor for arcs passed over and the arcs to be passed over of the *Śighra* anomaly for the heliacal rising and setting, retrograde motion and the like. By means of this daily rate of increase of the *Śighra* anomaly are obtained the days passed over and to be passed over of any of the above phenomena.

Illustration.—(*a*) To find the apparent daily motion of Mars at 785 *Saka* year and 12 synodic months.

Mars' *Śighra* motion	= 59′ 8″
Mars' daily mean motion	= 31′ 26″
∴ the daily motion of the *Śighra* anomaly	= 27′ 32″

The tabular differences at the 1st operation = 12°, for the interval of 32°

* Brahmagupta in the *Uttara Khaṇḍakhādyaka*, Ch. 1, however, gives methods of interpolation to make up for his rough tables here.

∴ the increase of the *Śighra* equation for 27′ 32″,

$$= \frac{27' \, 32'' \times 12}{32} = 10' \, 19''.$$

This halved = 5′ 10″.

As this is obtained from the positive tabular difference of 12, this last result is applied positively to the mean motion.

The mean daily motion as rectified by the first operation ∴

= 31′ 26″ + 5′ 10″ = 36′ 36″.

In the second operation, the tabular difference is 5′ for the interval of 900′, hence the apparent daily motion as corrected by the second operation

$$= 36' \, 36'' + \frac{5}{2} \times \frac{36' \, 36'' \times 5}{900}$$, as the mean anomaly lies in the third quadrant,

= 37′ 55″.

In the third operation, the tabular difference is also 5′ for the interval of 900′, hence the apparent daily motion as corrected by the third operation,

$$= 31' \, 26'' - 5 \times \frac{37' \, 55'' \times 5}{900}$$ as the mean anomaly lies in the fourth quadrant.

= 30′ 23″.

∴ the daily motion in *Śighra* anomaly = 59′ 8″ − 30′ 23″
= 28′ 45″; this will be used later on for finding the time of heliacal risings and other phenomena.

The increase of the *Śighra* equation for 28′ 45″

$$= \frac{28' \, 45'' \times 12}{32} = 10' \, 47''.$$

As this is obtained from the positive tabular difference of 12, it is applied positively to the rectified motion of the third operation. Thus the apparent daily motion of Mars = 30′ 23″ + 10′ 47″.

= 41′ 10″.

(*b*) From Mars' longitude as calculated before, *viz*., 0 sign 16° 52′ 35″, and the mean sun's longitude which is 11 signs 19° 11′ 19″ it is seen that the elongation from the mean sun = 27° 41′ 16″, again the elongation for setting = 17°. It is proposed to calculate in how many days more will Mars set heliacally. The method suggested by Brahmagupta is this:—

The *Śīghra* anomaly at the fourth operation $=$ 10 signs 14° 53′ 59″,
the *Śīghra* anomaly for setting in the west $=$ 11 signs 2°,
∴ the *Śīghra* anomaly to increase by $= 17° \; 6' \; 1''$,
rate of the *Śīghra* anomaly $= 28' \; 45''$;

∴ the time after which Mars will set $= \dfrac{17° \; 6' \; 1''}{28' \; 45''}$ days

$=$ 35 *da.* 41 *gh.* 21 *bin.*

This finishes the second chapter of the Khaṇḍakhādyakakaraṇa which relates to the mean and apparent longitudes and daily motions of the 'star planets.'

CHAPTER III

On the Three Problems relating to Diurnal Motion.

1. 159 divided by 16, 65 divided by 8, and 10 divided [by] 3; multiplied severally by the equinoctial shadow [i.]e., of a stick 12 units high) are the *binādīs* of half the [va]riation of a day from 30 *ghaṭikās* at the end of the 1st, [2]nd, and 3rd signs of the zodiac respectively.

The term equinoctial shadow means the length of shadow on [th]e horizontal plane of a stick of 12 digits set up vertically on the [gr]ound, at noon of the day on which the sun is at either of the [eq]uinoxes. If ϕ be the latitude of the station, it is taken $= 12$ [ta]n ϕ, although this should strictly be $= 12 \tan (\phi - \text{sun's semi-}$[dia]meter), if the shadow be measured by the umbra.

The arithmetical figures of this stanza have already been [co]nsidered in Chapter I, stanza 21. It now remains to give here the [rat]ionale of the rule:—

$$\sin (\tfrac{1}{2} \text{ variation of a day}) = \frac{R \sin \delta \times 12 \tan \phi}{12} \times \frac{R}{R \cos \delta}.$$

[A]ccording to Indian astronomers, half the variation of a day, [i]n the diurnal circle between the horizon and the six o'clock [circl]e (i.e., the great circle passing through the celestial poles and [e]ast and west points). The 'sine' of this arc in the diurnal circle [is ca]lled *Kujyā*; which reduced to the great circle becomes the [sine] of *cara* or ½ variation of a day.*

[F]or arriving at the equation given above, two similar right-angled [trian]gles are used.

[T]he first of these is thus constructed on the armillary sphere:— [From th]e point of intersection of the diurnal circle of the sun and the [six o']clock circle, let two perpendiculars be drawn, one on the line

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* Bhāskara II, *Gola*, vii, 1.

of intersection ... on the east-west ... perpendiculars consti... the two sides of a right-an... the first is called the *kujyā* or 'earth sine,' the second is ... between the diurnal circle and the celestial equator and is equal to the 'sine' of the declination. The third side which is on the horizon is equal to the distance between east-west line and the line of intersection of the diurnal circle and the horizon; this side is equal to the 'sine' of the amplitude and called the *agrā*. This triangle is thus stated by Brahmagupta in *Brāhmasphuṭa-siddhānta*, xvi, 61. "The base is the 'sine' of the declination, the perpendicular is the *kṣitijyā* (or *kujyā*) and the square root of the sum of the squares is the *agra* or the 'sine' of the amplitude."

Of this triangle one acute angle is equal to the angle between the horizon and the six o'clock circle, and is equal to the latitude ϕ of the observer.

The second triangle has its base equal to the gnomon, 12 digits high, and the perpendicular equal to the equinoctial midday shadow of the same gnomon. This triangle has one acute angle $= \phi$, the latitude of station.

From the above two similar right-angled triangles, we have

kujyā : $R \sin \delta$ = Equinoctial shadow : 12, where δ = sun's declination.

$$\therefore \ kujy\bar{a} = \frac{R \sin \delta \times \text{Equinoctial shadow}}{12}$$

Now this *kujyā* is a 'sine' in the diurnal circle of radius $= R \cos \delta$. It becomes the *carajyā*, when reduced to the great circle.

$$\therefore \ R \sin (cara) = \frac{R \sin \delta}{12} \times \text{Equinoctial shadow} \times \frac{R}{R \cos \delta}$$

2. Multiply the number of minutes of the daily motion of a planet by the number of *binādīs* of ascensional difference or *cara* and divide by 3600 (*i.e.*, the number of *binādīs* in a whole day); apply the resulting minutes negatively to planets at mean sunrise and positively to planets at mean sunset, when the sun is in the northern hemisphere and in the inverse order when the sun is in the southern hemisphere.

This stanza has been already explained in Chapter I. See stanza 22 of Chapter I.

the ascensional difference when the sun is in the northern hemisphere and respectively increased and decreased when the sun is in the southern hemisphere, doubled will give the lengths of the night and the day in *ghaṭikās*.

This is the same as the 23rd stanza on Chapter I.

4. The durations in *binādīs* of the risings of the first three signs of the zodiac at Laṅkā (*i.e.*, on the equator), are 278, 299, and 323. These diminished by the *binādīs* of local ascensional difference are the durations of the risings of the three signs at one's own place. The figures written in the reverse order and increased by the ascensional difference in the reverse order are the durations of the risings of the next three signs at the observer's place.

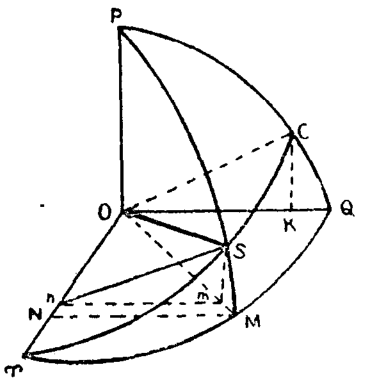
The meaning of this stanza is presented in a tabular form:

Sign	Duration in *binādīs* of the rising of the sign on the equator	*Binādīs* of the corresponding ascensional difference at Kurukṣetra acc. to Pṛthūdaka	Duration of the rising of the sign at Kurukṣetra in *binādīs*
Aries ...	278	−69	209
Taurus ...	299	−57	242
Gemini ...	323	−23	300
Cancer ...	323	+23	346
Leo ...	299	+57	356
Virgo ...	278	+69	347
Libra ...	278	+69	347
Scorpio ...	299	+57	356
Sagittarius	323	+23	346
Capricorn	323	−23	300
Aquaris ...	299	−57	242
Pisces ...	278	−69	209

The *binādīs* of ascensional difference for each sign of the zodiac have already been considered in Chapter I, 21. It is first necessary to explain how the durations in *binādīs* of the risings of the signs on the equator are obtained.

The method is thus expressed by Bhāskara in his *Grahagaṇita, Spaṣṭādhikāra*, 54, 55, Commentary, thus:—

"The 'sine' of the last point of Aries in the ecliptic is the hypotenuse, the 'sine' of the declination of the same point is the perpendicular........., the square root of the difference of their squares is the base and is a 'sine' in the diurnal circle of the same point. In this way the 'sine' of two signs is the hypotenuse and the 'sine' of the declination of the same is the perpendicular, and the square-root of the difference of their squares is a 'sine' in the diurnal circle of the last point of Taurus. In the same way the 'sine' of three signs is the hypotenuse, the 'sine' of the maximum declination (of the sun) is the perpendicular and the smallest radius of the sun's diurnal circle is the base. These bases are reduced to the great circle. These bases are multiplied by the radius and divided by the radii of the respective diurnal circles and taken as 'sines' of the arcs, of which the first represents the duration of the rising of Aries, the second of the first two signs, the third of the first three signs"



Let O be the centre of the armillary sphere; P, the north celestial pole; γMQ, the celestial equator; γSC the ecliptic, $\gamma C = \gamma Q = 90°$, PSM a secondary to the equator. If $\gamma S = l$, $SM = \delta$, the declination of S; Sn = 'sine' of l. Here nm is the sine of γM in the diurnal circle of S. Here $\angle Snm = \angle COK = w$, the obliquity of the ecliptic; $CK = R \sin w$, $OK = R \cos w$, $Sm = R \sin \delta$, $Sn = R \sin l$.

$$\therefore \quad Sm : Sn = CK : CO,$$

$$\text{or } R \sin \delta = \frac{R \sin l \times R \sin w}{R} \quad \ldots \quad \ldots \quad \ldots$$

Again $nm : nS = OK : OC$,

$$\therefore \quad nm = \frac{R \sin l \times R \cos w}{R};$$

$$\text{Now } R \sin \gamma M = \frac{nm}{R \cos \delta} \times R,$$

$$\therefore \quad R \sin \gamma M = \frac{R \sin l \times R \cos w}{R} \times \frac{R}{R \cos \delta}$$

or $R \sin R.A. = \dfrac{R \sin l \times R \cos w}{R \cos \delta}$, where $R.A.$ stands for the arc γM ... ... (2)

Now the values of δ at the end of 1, 2, and 3 signs are respectively *:—

	Acc. to Brahmagupta	As worked out accurately
$\delta_1 =$	703′	704′
$\delta_2 =$	1238′	1238′
$\delta_3 =$	1440′	1440′

And the values of $R.A.$ for 1, 2, and 3 signs as calculated are respectively *:—

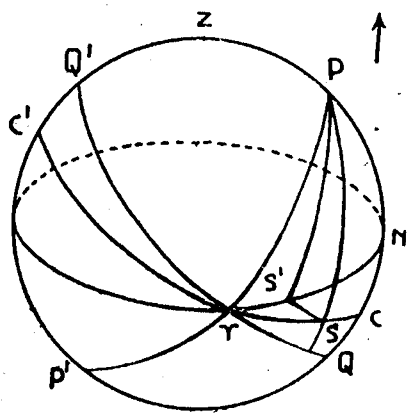
	Calculated values	Differences	Differences in *binādis*	As given by Brahmagupta
$R.A_1.=$	27° 49′	27° 49′	278	278
$R.A_2.=$	57° 43′	29° 54′	299	299
$R.A_3.=$	90°	32° 17′	323	323

This proves Brahmagupta's rule.

Now we consider the time interval for the rising of the arc l at a place of which the latitude is ϕ.

* The obliquity of the ecliptic has been taken to be 24°, as in all Hindu astronomical works.

Let $NPZQ'$ be the observer's meridian ; N, the north point ; P, the celestial pole ; Z, the zenith ; Q', the culminating point of the equator ; $NS'r$, the horizon ; r, the east point, to which the first point of r coincides ; $CSrC'$, the ecliptic. The arc $rS=l$. PSM is the secondary to the equator ; SS' a part of the diurnal circle of S. The time in which rS rises above the horizon is represented by $\angle SPS'$ in the figure.



Now $\angle SPS' = rPS - rPS'$

$= rPS - (S'PQ' - Q'Pr)$

$= rPS -$ (half day $-$ 6 hours)

$= rPS -$ ascensional difference for S.

Here rPS represents the time in which l length of arc rises on the equator. Now by putting $l = 1$, 2, or 3 signs, we have the local intervals (sidereal) for the risings of 1 sign, 2 signs and 3 signs. Now each of these subtracted from the next gives the durations in which the signs of the zodiac severally rise at any place. Thus is proved the second half of the stanza.

5. The sun (*i.e.*, the sun's longitude) increased in proportion, from the time in *ghaṭikās* elapsed since sunrise, by means of local time durations for the rising of the signs of the zodiac, becomes the orient ecliptic point ; again by making the sun (*i.e.*, the sun's longitude) equal to the orient ecliptic point by the local time intervals for the rising of the signs, is found the time elapsed since sunrise.

Illustration.—(*a*) To find the orient ecliptic point at 5 *ghaṭikās* elapsed since sunrise at Kurukṣetra ; the sun's longitude being at that instant= 11 signs 19° 46′ 36″.

Here the remainder of Pisces =10° 13′ 24″= 613′ 24″.

The time in which Pisces rises is 209 *bināḍīs*,

$\therefore$ the time in which 613′ 24″ rises $= \dfrac{209 \times 613' \, 24''}{30 \times 60}$ *bināḍīs*

$= 71 \cdot 222$ *bināḍīs*

$= 71$ *bināḍīs*.

The time elapsed since sunrise

$= 5$ *ghaṭikās* $= 300$ *bināḍīs*.

Now from 300 *bināḍīs*,

subtract 71 *bināḍīs* for the residue of Pisces,

also 209 *bināḍīs* for the sign Aries,

$\therefore$ 20 *bināḍīs* of the duration of the rising of Taurus is also passed;

and 20 *bināḍīs* of Taurus corresponds to $\dfrac{20 \times 30}{242}$ degrees or 2° 28′ 45″.

Now Sun + residue of Pisces + Aries + 2° 28′ 45″

= 1 sign 2° 28′ 45″, which is the longitude of the orient ecliptic point.

(*b*) To find the time elapsed since sunrise when the longitude of the orient point is 1 sign 2° 28′ 45″, and the sun's longitude is 11 signs 19° 46′ 36″.

Now, 2° 28′ 45″ of Taurus corresponds to 20 *bināḍīs*, by the converse process.

Aries corresponds to 209 *bināḍīs*.

The residue of Pisces, to 71 *bināḍīs* by the converse process,

$\therefore$ the time elapsed since sunrise = 300 *bināḍīs*,

= 5 *ghaṭikās*.

(*c*) To find the orient ecliptic point at 3 *ghaṭikās* before sunrise at Kurukṣetra, the sun's longitude being 11 signs 19° 38′ 39″.

Now 19° 38′ 39″ of Pisces correspond to

$\dfrac{19° \; 38' \; 39''}{30°} \times 209$ *bināḍīs* $= 137$ *bināḍīs*.

3 *ghaṭikās* $= 180$ *bināḍīs* ;

$\therefore$ difference $= 43$ *bināḍīs*.

Now 43 *bināḍīs* of Aquaris $= \dfrac{43 \times 30°}{242} = 5° \; 19' \; 50''$;

∴ the longitude of the orient ecliptic point

$= \text{Sun} - 19° \; 38' \; 39'' - 5° \; 19' \; 50''$

$= 11 \text{ signs } 19° \; 38' \; 39'' - 19° \; 38' \; 39'' - 5° \; 19' \; 50''$

$= 10 \text{ signs } 24° \; 40' \; 10''$.

6. Thirty increased severally by nine, six and one; twenty-four, fifteen and five are the tabular differences of 'sines' at intervals of half a sign. For any arc, the 'sine' is the sum of the parts passed over increased by the proportional part of the tabular difference to be passed over.

As explained already in Chapter I, the stanza means, 150 sin 15° = 39, 150 sin 30° = 75, 150 sin 45° = 106, 150 sin 60° = 130, 150 sin 75° = 145 and 150 sin 90° = 150.

The method of arriving at the 'sines' is this—

'Sine' of 30° = 150 sin 30° = 75

'Sine' of 60° $= \sqrt{150^2 - 75^2} = \sqrt{16875} = \frac{\sqrt{16875 \times 625}}{25} = \frac{3248}{25}$

$= 129 \frac{23}{25} = 130,$

∴ 'vers' 30° = 150 − 130 = 20.

'Sin' 15° $= \frac{1}{2}\sqrt{75^2 + 20^2} = 38\frac{21}{25} = 39.$

∴ 'Sine' of 75° $= \sqrt{150^2 - \frac{6025}{4}}$

$= 144 \frac{22}{25} = 145.$

Again 'sine' 45° $= \sqrt{\frac{150^2}{2}}$

$= 106 \frac{2}{25} = 106.$

The above method is that of *Brāhmasphuṭa-siddhānta* xxi, 20-21. The method of extracting the square root is given by Śrīdhara, in his *Triśatikā*, rule 46. This must have been known to Āryabhaṭa and Brahmagupta and used for calculating the 24 'sines' in a quadrant.

7. The declinations (for each half a sign) in minutes are 362, 703, 1002, 1238, 1388 and 1440, increased or decreased by the planet's celestial latitude, according as they have the same or the opposite denominations.

This stanza is the same as the 29th stanza of Chapter I. The explanation, etc., of the stanza have been given there. The method of finding the declination for any value of the sun's longitude has also been discussed in the exposition of stanza 4 of this chapter.

8. Diminish or increase the latitude of the place by the declination of the sun (according as he is north or south of the equator); call the result *anaṣṭa* (*i.e.*, not disturbed); the 'sine' of the arc got by subtracting the *anaṣṭa* from 90 is the divisor of the 'sine' of the *anaṣṭa* multiplied by 12. The quotient is the length of the shadow of the gnomon at noon.

If ϕ and δ be the latitudes of the observer and the sun's north declination respectively, then

$\phi - \delta$ = *anaṣṭa* or the sun's meridian zenith distance.

The rule says that the shadow of the gnomon

$= \frac{R \sin(\phi - \delta) \times 12}{R \sin\{90° - (\phi - \delta)\}} = 12 \tan(\phi - \delta)$, the usual height of the gnomon being 12 digits.

9. The radius divided by the 'sine' of the complement of the sum or difference of latitude and the declination, and multiplied by 12, is the length of the hypotenuse (*i.e.*, the line joining the top of the gnomon to the end of its noon shadow) in digits, etc.

If this hypotenuse be denoted by h,

then $h = \frac{R \times 12}{R \cos(\phi - \delta)} = 12 \sec(\phi - \delta)$, where δ is north.

The rule as the previous one, is evident.

10. Take the square roots of the squares of the hypotenuse and of the shadow respectively decreased and increased

by square of *śaṅku* (*i.e.*, 144) ; the results are respectively the shadow and the hypotenuse. Half the day diminished by the hour angle is either the part of the day elapsed or the remaining part of the day.

The meaning of this stanza is clear.

11. Put down the radius in two places, divide in each place by the hypotenuse of the equinoctial shadow, multiply in the two places by *śaṅku* (*i.e.*, 12) and the equinoctial shadow. The two results are respectively the 'sines' of the colatitude and the latitude. The arc of the 'sine' of the latitude is the latitude of the observer.

Let the equinoctial shadow be denoted by P, and the equinoctial hypotenuse by H. Then $P = 12 \tan \phi$, $H = 12 \sec \phi$. The stanza says that $R \cos \phi = \frac{R \times 12}{H}$ and $R \sin \phi = \frac{R \times P}{H}$.

12. Subtract as many parts as possible of the tabular differences of the 'sines' from the given 'sine'; multiply the remainder by 900, and divide by the tabular difference that cannot be subtracted ; add the resulting minutes to 900′ multiplied by the number of tabular differences passed over ; the final result will be the arc corresponding to the given 'sine.'

A rule for finding the arc when the sine is known, is already explained in Chapter I, 32.

13. The radius increased or decreased by the 'sine' of half the variation of the day (*i.e.*, sine of *cara*), is called *antyā* or the 'versed sine' of half the day. *Antyā* diminished by the 'versed' sine of the hour angle of the sun is the divisor of the *antyā* multiplied by the midday hypotenuse. The quotient is the hypotenuse of the gnomon triangle at the desired time.

According to the rule,

$$antyā = R \pm R \sin (cara),$$
$$= R \text{ vers half the day.}$$

Hypotenuse at any desired time,

$$= \frac{antyā \times \text{midday hypotenuse}}{antyā - R \text{ vers } h}, \text{ where } h \text{ is the hour angle.}$$

The truth of the rule may be seen thus:—Let z be the sun's zenith distance at the time, ϕ and δ, the latitude of the observer and the sun's declination respectively.

Then, the hypotenuse at any desired time $= 12 \sec z$.

The midday hypotenuse $= 12 \sec (\phi - \delta)$, since $(\phi - \delta)$ is equal to the meridian zenith distance.

We are to show that

$$12 \sec z = \frac{\left(R + \frac{R \times R \sin \delta \times R \sin \phi}{R \cos \delta \times R \cos \phi}\right) 12 \sec (\theta - \delta)}{\frac{R \times R \sin \delta \times R \sin \phi}{R \cos \delta \times R \cos \phi} + R \cos h}$$

or $$\frac{12}{\cos z} = \frac{12}{\sin \delta \sin \phi + \cos \delta \cos \phi \cos h}, \text{ which is evident.}$$

Now we turn to the method by which the rule was obtained by Brahmagupta.

If in the armillary sphere, from any point of the sun's diurnal circle a perpendicular be drawn to the line of intersection of the horizon and the same diurnal circle, this perpendicular is called *iṣṭahṛti* ; the perpendicular from the same point of the diurnal circle, on the horizon is called *śaṅku*. The line joining the feet of these perpendiculars lying on the horizon is called *śaṅkutala*. In this triangle, which is right-angled the angle opposite to the *śaṅkutala* is $= \phi$, the latitude of the station. Again if from the point of intersection of the diurnal circle and the meridian, a perpendicular be drawn to the line of intersection of the diurnal circle and the horizon, it is called *cheda* ; the perpendicular from the same point of diurnal circle on the horizon is called the midday *śaṅku* ; here also the line joining the feet of the two perpendiculars on the north-south line is the third side of a right-angled triangle of which also one angle is ϕ.

From the above two similar triangles, we get the proportion :—

$$\frac{\text{midday } \acute{s}a\dot{n}ku}{cheda} = \frac{\acute{s}a\dot{n}ku}{i\d{s}\d{t}ah\d{r}ti},$$

$$\therefore \quad \acute{s}anku = \frac{i\d{s}\d{t}ah\d{r}ti \times \text{midday } \acute{s}a\dot{n}ku}{cheda}.$$

If R be the radius of the armillary sphere, then,

R : *śanku*= Hypotenuse of shadow : 12 ;

$$\therefore \quad \text{hypotenuse of shadow} = \frac{12 \times R}{\acute{s}a\dot{n}ku} = \frac{12 \times R \times cheda}{i\d{s}\d{t}ah\d{r}ti \times \text{midday } \acute{s}a\dot{n}ku}.$$

$$\text{Again } \frac{\text{hyp. of midday shadow}}{12} = \frac{R}{\text{midday } \acute{s}a\dot{n}ku},$$

$$\therefore \quad \text{hypotenuse of shadow} = \frac{\text{hyp. of midday shadow} \times cheda}{i\d{s}\d{t}ah\d{r}ti}.$$

Now *cheda* is readily seen from the armillary sphere to be

= versed sine of half day in the diurnal circle,

$$= \frac{R \cos \delta}{R}(R + R \sin cara) = \frac{R \cos \delta}{R} \times anty\bar{a}.$$

Also *iṣṭahṛti*

= *cheda* − versed sine of the hour angle in the diurnal circle,

$$= \frac{R \cos \delta}{R}(R + R \sin cara - R + R \cos h),$$

$$= \frac{R \cos \delta}{R}(anty\bar{a} - R \text{ vers } h) ;$$

∴ hypotenuse of the shadow at any time,

$$= \frac{\text{hypotenuse of midday shadow} \times anty\bar{a}}{anty\bar{a} - R \text{ vers } h},$$

which proves Brahmagupta's rule.

14. Or, the divisor is obtained thus :—Subtract from the time elapsed since sunrise in the forenoon or from the time to the sunset in the afternoon, the half the variation of the day or the ascensional difference ; of the resulting arc take the sine and increase it by the sine of the *cara* or the ascensional difference, the final result is the same divisor.

This is easily seen in the armillary sphere.

15. Multiply the *antyā* by the hypotenuse of the midday shadow and divide by the hypotenuse of the shadow at any given time ; then from the *antyā* subtract the quotient obtained, the arc of the remainder taken as the versed 'sine,' represents the *asus* (=4 sec. of time) of the incline from the noon (*i.e.*, of the hour angle).

If the 'versed sine' of an arc be given, the arc itself is obtained from the series of tabular differences of sines by using it in the inverse order. This explains the meaning of the word 'versed sine.'

Brahmagupta's tabular differences of sines being

39, 36, 31, 24, 15 and 5, the tabular differences of versed sines are,

5, 15, 24, 31, 36 and 39 ; the sines are,

39, 75, 106, 130, 145 and 150 ; and the versed sines

5, 20, 44, 75, 111 and 150.

The above stanza gives the rule for finding the hour-angle when the shadow of the gnomon or the hypotenuse of the gnomon-shadow triangle is known.

If h be the hour angle, it says that

$$R \text{ vers } h = anty\bar{a} - \frac{anty\bar{a} \times \text{midday hypotenuse}}{\text{Hypotenuse of the shadow}}.$$

The truth of the rule is seen thus :—

We should have,

$$R \text{ vers } h = \left(R + \frac{R \times R \sin \delta \times R \sin \phi}{R \cos \phi \times R \cos \delta} \right) \left(1 - \frac{\text{midday hypotenuse}}{\text{hypotenuse of shadow}} \right)$$

$$= \frac{R \cos (\phi - \delta)}{\cos \phi \cos \delta} \times \left(1 - \frac{12 \sec (\phi - \delta)}{12 \sec z} \right)$$

$$\text{or } R \text{ vers } h = \frac{R\cos(\phi-\delta)}{\cos\phi\cos\delta}\left\{\frac{\cos(\phi-\delta)-\cos z}{\cos(\phi-\delta)}\right\},$$

$$= \frac{R\{\cos(\phi-\delta)-\cos z\}}{\cos\phi\cos\delta},$$

or $(1-\cos h)\cos\phi\cos\delta = \cos(\phi-\delta)-\cos z$,

or $\cos z = \sin\phi\sin\delta+\cos\phi\cos\delta\cos h$, which is evident.

We are now to consider how this relation was obtained.

As in the exposition of the previous stanza, we have,

$$\frac{\text{midday } \acute{s}a\dot{n}ku}{cheda} = \frac{\acute{s}a\dot{n}ku}{i\d{s}\d{t}ah\d{r}ti},$$

$$\therefore\ i\d{s}\d{t}ah\d{r}ti = \frac{cheda\times\acute{s}a\dot{n}ku}{\text{midday } \acute{s}a\dot{n}ku}.$$

Again, *śaṅku* : R = 12 : hypotenuse of shadow, and

R : midday *śaṅku* = noon hypotenuse : 12,

∴ *śaṅku* : midday *śaṅku* = noon hypotenuse : hyp. of shadow;

$$\therefore\ i\d{s}\d{t}ah\d{r}ti = \frac{cheda\times\text{noon hypotenuse}}{\text{hyp. of shadow}}.$$

Now *cheda*–*iṣṭahṛti*

= versed sine of the hour angle in the diurnal circle ;

∴ versed sine of the hour angle in the great circle

$$= (cheda - i\d{s}\d{t}ah\d{r}ti)\times\frac{R}{R\cos\delta}$$

$$= anty\bar{a} - \frac{anty\bar{a}\times\text{noon hypotenuse}}{\text{hypotenuse of the shadow}},$$ which is Brahmagupta's rule. This stanza and the next are taken from *Brāhmasphuṭa-siddhānta*, iii, 44-45.

16. If the same quotient diminished or increased by the 'sine' of the ascensional difference according as the sun is on the north or south of the celestial equator, be taken as the 'sine,' the arc of the same increased or decreased by the ascensional difference (*i.e.*, ½ the variation of the day from 30 *ghaṭikās* or 12 hours) is the time elapsed of the day in the forenoon or the time to sunset in the afternoon.

The truth of the rule is seen from the following steps :—

iṣṭahṛti–*kujyā* = sine of the complement of the hour angle in the diurnal circle ;

∴ the 'sine' of the complement of the hour angle

$$= \frac{R}{R\cos\delta}(i\d{s}\d{t}ah\d{r}ti - kujy\bar{a}),$$

$$= \frac{anty\bar{a}\times\text{noon hypotenuse}}{\text{hypotenuse of the shadow}} - R\sin(\text{ascensional difference}).$$

The rest requires no explanation.

Illustrations :

(1) Given that the longitude of the sun at noon at Kurukṣetra 11 signs 19° 53′ 24″, to find the noon shadow and the noon hypotenuse (stanza 8).

The sun's south declination = 4° 5′ 25″.

Again at Kurukṣetra, the equinoctial shadow = 7 digits ;

∴ the latitude of Kurukṣetra = 30° 15′ 33″ ;

∴ the sun's meridian zenith distance

= 34° 20′ 58″ ;

its complement = 55° 39′ 2″ ;

'sine' of 34° 20′ 58″ = 150 × sin 34° 20′ 58″

= 84^p 38′.

'sine' of 55° 39′ 2″ = 150 × sin 55° 39′ 2″ = 123^p 50′.

$$\text{The noon shadow} = \frac{12\times\text{'sine' } 34^\circ\,20'\,58''}{\text{'sine' } 55^\circ\,39'\,2''}$$

$$= \frac{12\times 84^p\,38'}{123^p\,50'} = 8 \text{ digits } 14';$$

$$\text{the noon hypotenuse} = \sqrt{12^2+(8^p\,14')^2}$$

= 14 digits 33′.

(2) To calculate the noon hypotenuse by the rule of stanza 8.

The complement of the sum of the latitude of the station and the sun's south declination

= 55° 39′ 2″.

Its sine = 123^p 50′ ;

$$\therefore \text{ the noon hypotenuse} = \frac{150^p\times 12}{123^p\,50'} = 14 \text{ digits } 33'.$$

(3) To calculate the length of the hypotenuse of the shadow at 5 *ghaṭikās* after sunrise (stanza 13).

The sun's south declination $= 4° \ 5' \ 25''$;

$\therefore$ the 'sine' of the ascensional difference

$$= 150 \tan \phi \tan \delta = 6^p \ 15'.$$

As the sun is south of the equator,

R – 'sine' (ascensional difference)

$= 150^p - 6^p \ 15' = 143^p \ 45'$; here this is the *antyā*.

Half the day $= 15$ *gh.* -23 *bin.* $= 14$ *gh.* 37 *bin* ;

time elapsed since sunrise $= 5$ *gh.* ;

$\therefore$ the hour angle $= 9$ *gh.* 37 *bin.*

$= 577$ *bin.*

$= 3462$ min. of arc.

'Versed sine' of $3462' = 69^p \ 52'$.

Now *antyā* $- R$ vers $h = 143^p \ 45' - 69^p \ 52'$

$= 73^p \ 53'$;

$\therefore$ the required hypotenuse of the shadow

$$= \frac{\text{antyā} \times \text{hypotenuse at noon}}{\text{antyā} - R \text{ vers } h},$$

$$= \frac{143^p \ 45' \times 14 \text{ digits } 32'}{73^p \ 53'},$$

$$= \frac{2089^p \ 10'}{73^p \ 53'} = \underline{28 \text{ digits } 16'} ;$$

$\therefore$ shadow $= \sqrt{(28\tfrac{16}{60})^2 - 144}$

$= \underline{25 \text{ digits } 7'}$.

(4) To find the time elapsed since sunrise at Kurukṣetra, when the shadow of the gnomon $= 25$ digits $7'$; the sun's south declination being $4° \ 5' \ 25''$, the latitude of Kurukṣetra $30° \ 15' \ 33''$ (stanza 15).

Here the *antyā*, as before $= 143^p \ 45'$;

the noon hypotenuse $= 14$ digits $32'$;

the shadow $= 25$ digits $7'$;

the hypotenuse of the shadow $= \sqrt{12^2 + (25\tfrac{7}{60})^2}$,

$= 28$ digits $16'$;

the 'versed sine' of the hour angle

$$= \text{antyā} - \frac{\text{antyā} \times 14^p \ 32'}{28^p \ 16'},$$

$$= \text{antyā} \left(1 - \frac{14^p \ 32'}{28^p \ 16'}\right),$$

$$= 143^p \ 45' \times \frac{13^p \ 34'}{28^p \ 16'} = 68 \ 47',$$

$=$ Versed sine of $57° \ 13'$ nearly;

$\therefore$ the hour angle $= 57° \ 13' = 3433'$

$= 572$ *bināḍīs*,

$= 9$ *ghaṭikās* 32 *bināḍīs*.

Half the day $= 14$ *ghaṭikās* 37 *bināḍīs*.

$\therefore$ the time elapsed since sunrise $=$ 5 *ghaṭikās* 5 *bināḍīs*.

Here 5 *bināḍīs* is an error of calculation. No illustration of the 16th stanza appears to be necessary.

Precession of the Equinoxes.—In all calculations at the time of Pṛthūdaka (786 Śaka) no allowance was made for the shifting of the equinoxes in finding the declination. However out of date and incorrect may be the astronomical constants of the *Khaṇḍakhādyaka*, still, should one use them now-a-days for any practical calculations, one has to make allowance for the precession of the equinoxes. It is therefore necessary to consider this topic here and to find out a fairly accurate mean rate of precession.

The length of the year in the *Khaṇḍakhādyaka*,

$$= \frac{1577917800}{4320000} \ da. = \frac{292207}{800} \ da.$$

$= 365{\cdot}25875$ *da.*

The mean length of the tropical year

$= 365{\cdot}24219879$ *da.* (Newcomb)

The excess of the *Khaṇḍakhādyaka* year

$= {\cdot}01655121$ *da.*

The sun's motion in ${\cdot}01655121$ days,

$= 59' \ 8''{\cdot}19 \times {\cdot}01655121$ days

$= 58''{\cdot}72685$ which should be taken as the mean value of the precession for the *Khaṇḍakhādyaka* year.

Taking the vernal equinox of the 421 of the *Śaka* era (*i.e.*, Āryabhaṭa's time) to be the beginning of the first point of *Aświni nakṣatra*, at 800 of the Śaka era the total precession would be

$$6° \ 40' \ 59''.$$

Pṛthūdaka at this time observed the value of the total precession to be 6° 30′, which must be considered as fairly accurate.*

At the Śaka year 1072, the total precession would be 10° 37′ 11″.

Bhāskara, in 1072 Śaka year, observed it to be 11°, which also must be considered as fairly accurate.†

Again at 1851 of the Śaka year the total amount of precession would be = 23° 19′ 30″.

Hence we can take 23° 19′ 30″, to be the total amount of the precession at 1851 of the Śaka era in our subsequent calculations relating to the present times.

It is perhaps not out of place to record here that the followers of Āryabhaṭa, from Sūryadeva Jajvan's time, adopted the value of the precession at $\frac{1}{61}$th of a degree, *i.e.*, 59″·504 per year.‡ According to Mañjula (932 A.D.) the mean rate of precession is about 59″·9 per year.§ According to Viṣṇucandra as quoted by Pṛthūdaka in his Comm. on the *Brāhmasphuṭa-siddhānta*, x, 54, the mean annual rate of precession is 56″·8233 per year.

The current *Siddhāntas* which are of unknown origin and the date of almost all of which, must be after the time of Brahmagupta, accept the mean rate to be 54″ per year.

This brings us to the end of Chapter III, relating to the three problems of the orient ecliptic point, the shadow and the local time.

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* Amarāja's Commentary in Pandit Bābuā Miśra's edition, p. 108.

† *Grahagaṇita, Pātādhikāra*, Comm. on 3-6.

‡ Parameswara's Comm. on *Āryabhaṭīya, Kālakriyā*, 10.

§ Bhāskara's *Gola*, 18.

CHAPTER IV

On Lunar Eclipses.

1 (1st half). The sun and the moon are made equal in respect of minutes, etc., by being decreased or increased by the result of interpolation from the *ghaṭikās* which show the end of the *tithi* (*i.e.*, opposition), whether elapsed or to come.

As this chapter relates to the lunar eclipse, we start with the calculation after the *Khaṇḍakhādyaka*, of the total lunar eclipse on the 3rd April, 1931 A.D. The time according to the Śaka era is 1852, 2 synodic months and 15 *tithis*. Hence the *ahargaṇa* = 462404, on Thursday the 2nd April at midnight at Ujjayinī.

The mean sun	= 11 signs 17° 9′ 27″.
The mean moon	= 5 signs 17° 12′ 42″.
The sun's apogee	= 2 signs 20° 0′ 0″.
The moon's apogee	= 11 signs 4° 50′ 39″ − 5″,
	= 11 signs 4° 50′ 34″.
The sun's mean anomaly	= 8 signs 27° 9′ 27″ ;
∴ the sun's equation	= + 134′ sin 87° 9′ 27″,
	= 2° 13′ 50″ ;
∴ the sun's apparent longitude	= 11 signs 19° 23′ 17″.
The moon's mean anomaly	= 6 signs 12° 22′ 8″ ;
∴ the moon's equation	= + 296′ sin 12° 22′ 8″,
	= + 1° 3′ 24″ ;
∴ the moon's apparent longitude	= 5 signs 18° 16′ 6″.
The sun's apparent daily motion	= 59′ 8″ + 20″ = 59′ 28″.
The moon's apparent daily motion	= 790′ 35″ + 67′ 9″ = 857′ 44″.

These are the apparent daily motions as calculated from the *Khaṇḍakhādyaka*; but if they are calculated from the rule of *Brāhmasphuṭa-siddhānta*, II, 41, they become respectively :—

The sun's apparent daily motion	= 59′ 8″ + 31″,
	= 59′ 39″.

The moon's apparent daily motion $= 790' 35'' + 65' 37''$,
$= 856' 12''$.

Here we adopt these latter values.

Now, Moon $=$ 5 signs $18° 16' 6''$.
Sun $=$ 11 signs $19° 23' 17''$.
Difference $=$ 5 signs $28° 52' 49''$.

The required difference for opposition
$=$ 6 signs $0° 0' 0''$;

∴ the difference to be made up by the moon
$= 1° 7' 11'' = 67' 11''$;

∴ the *ghaṭikās* after which this will happen

$$= \frac{67' 11'' \times 60}{856' 12'' - 59' 39''} \text{ ghaṭikās},$$

$=$ 5 *ghaṭikās* 5 *bināḍīs* 2 *bipalas* ;

i.e., at this interval of time after the mean midnight at Ujjayinī, the opposition will happen.

The sun's longitude at that time will be
$=$ 11 signs $19° 28' 30''$.

The moon's longitude will be $=$ 5 signs $19° 28' 30''$.

The time of opposition in Ujjayinī local time is thus 2 hours 2 min., which in Calcutta local time is 2 hrs. 52 min. 33 secs. (the longitude of Ujjayinī $= 75° 52'$ east, and that of Calcutta $88° 30'$ east); but on that date the 3rd of April the instant of opposition is 2 hours A.M., Calcutta time. Thus there is here a difference of 52 min. 33 secs.

Again at *ahargaṇa* 462404, the longitude of the ascending node according to the *Khaṇḍakhādyaka*, with Pṛthūdaka's and Lalla's * correction $=$ 11 signs $22° 34' 47''$;

Correction for 5 gh. 5 bin. $= -17''$;

∴ the longitude of the ascending node at the instant of opposition $=$ 11 signs $22° 34' 30''$.

The moon's longitude $=$ 5 signs $19° 28' 30''$.

The sun's longitude $=$ 11 signs $19° 28' 30''$.

1. (2nd half). Take the sine of the arc got by diminishing the moon by the node, multiply it by 9 and divide by 5 ; the quotient taken as minutes is the celestial latitude of the moon.

* Lalla speaks of a correction of $-96'$ to the node in every 250 years elapsed from 421 of *Śaka* era, the total correction being taken at $-8°$ for 5 integral cycles of 250 years.

This has already been explained in Chapter I. The rule is equivalent to this :—

The moon's celestial latitude $= 270'$ sin (Moon-Node).
In the above example, Moon $-$ Node
$=$ 5 signs $26° 54' 0''$;

∴ the moon's celestial latitude at the instant of opposition
$= 270'$ sin $3° 6' = 14' 31''$.

2. The sun and the moon's apparent daily motions respectively multiplied by 11 and 10, and divided by 20 and 247, are their apparent diameters in minutes. Multiply their apparent daily motions in minutes by 25 and 8 ; one sixtieth of the difference of the results (of multiplication) is the angular diameter of the shadow in minutes.

The first half of the stanza has already been explained in Chapter I. From this rule the apparent diameter of the moon at the instant of opposition under consideration

$$= \frac{10 \times \text{moon's apparent daily motion}}{247}$$

$$= \frac{10 \times 856' 12''}{247} = 34' 40''.$$

As to the second part, we get from the *Āryabhaṭīya*, *Gola*, 39, interpreted in our own way that the diameter of the shadow
$= 2$ (moon's horizontal parallax $-$ sun's semidiameter $+$ sun's horizontal parallax).

and hence it is

$$= 2\left(\frac{\text{Moon's apparent daily motion}}{15} - \frac{11 \times \text{sun's apparent motion}}{40} + \frac{\text{sun's apparent motion}}{15}\right),$$

by the 1st half of the stanza, and from the general view of the Hindu astronomers that horizontal parallax $= \frac{1}{15}$ the daily motion of a planet;

∴ the diameter of the shadow

$$= \frac{8 \times \text{moon's appt. daily motion} - \text{sun's appt. daily motion} \times 25}{60},$$

which proves the rule.

This stanza is almost the same as the 6th stanza of the *Brāhma-sphuṭa-siddhānta*, Chapter IV.

We now calculate the diameter of the shadow for the eclipse under our consideration.

∴ It is $= \frac{8 \times 856' \, 12'' - 25 \times 59' \, 39''}{60} = 89' \, 18''$.

3 (1st half). Subtract the moon's latitude from half the sum of the diameters of the obscured and obscuring bodies; the remainder represents the portion obscured by the shadow or the obscuring body.

Here the moon's celestial latitude $= 14' \, 31''$.

Moon's diameter	$= 34' \, 40''$.
Diameter of the shadow	$= 89' \, 18''$.
Sum	$= 123' \, 58''$;
half	$= 61' 59''$;

∴ the portion obscured $= 47' \, 28''$, which is greater than the moon's diameter, hence this is a case of total eclipse.

3 (2nd half). If the obscured part is greater than the obscured body itself, it is a case of total eclipse, if less, a case of partial eclipse.

4. Take the sum and the difference of the semi-diameters of the obscured and obscuring bodies; from the squares of the results, subtract the squares of the moon's latitude; from the square roots of the results are obtained in the same manner as of *tithis* half durations of the eclipse and of the total obscuration.

In the present case, the sum of the semi-diameters $= 61' \, 59''$.

Their difference $= 27' \, 19''$.

The moon's celestial latitude $= 14' \, 31''$.

∴ half duration of the eclipse by the rule

$$= \frac{\sqrt{(61' \, 59'')^2 - (14' \, 31'')^2}}{856' \, 12'' - 59' \, 39''} \times 60 \; ghaṭikās,$$

$= 4{\cdot}539033$ *ghaṭikās*

And half duration of total obscuration

$$= \frac{\sqrt{(27' \, 19'')^2 - (14' \, 31'')^2}}{856' \, 12'' - 59' \, 39''} \times 60 \; ghaṭikās,$$

$= 1{\cdot}74303$ *ghaṭikās*.

Thus are obtained the first approximations to the two ends of these two phases of the eclipse. The supposition here is that the moon's orbit is parallel to the ecliptic. The successive approximations are carried out as shown below, but the rationale is not clearly expressed.

The next step is to take up the beginning of the eclipse.

Now the half duration for the beginning of the eclipse $= 4{\cdot}539033$ *ghaṭikās*.

The moon's motion in this time	$= 64' \, 55''$.
Motion of the node in this time	$= 14''$.
The moon's longitude at this time	$= 5$ signs $18° \, 23' \, 35''$.
The node's longitude at this time	$= 11$ signs $22° \, 34' \, 44''$.
The moon's celestial latitude	$= 270' \sin 4° \, 11' \, 9''$,
	$= 19' \, 42''$.
The sum of the semi-diameters of the obscured and obscuring bodies	$= 61' \, 59''$.

Hence the half duration for the beginning of the eclipse

$$= \frac{\sqrt{(61' \, 59'')^2 - (19' \, 42'')^2}}{856' \, 12'' - 59' \, 39''} \times 60 \; ghaṭikās,$$

$= 4{\cdot}4268$ *ghaṭikās* which represents the *2nd approximation*.

Again the moon's motion in $4{\cdot}4268$ *ghaṭikās* $= 63' \, 10''$.

The motion of the node in $4{\cdot}4268$ *ghaṭikās* $= 14''$.

The moon's longitude at 4·4268 *ghaṭikās* before the opposition

$= 5$ signs $18° 25' 20''$.

The node's longitude $= 11$ signs $22° 34' 44''$.

The moon's celestial latitude

$= 270' \sin 4° 9' 24''$,

$= 19' 34''$.

The sum of the semi-diameters

$= 61' 59''$.

Hence the half duration for the beginning of the eclipse

$$= \frac{\sqrt{(61' 59'')^2 - (19' 34'')^2}}{856' 12'' - 59' 39''} \times 60 \text{ } ghaṭikās,$$

$= 4·430015$ *ghaṭikās*, which represents the *3rd approximation.*

Again the moon's motion in these 4·4300 *ghaṭikās*

$= 63' 13''$;

the node's motion in the same time

$= 14''$;

∴ the moon's longitude at 4·430 *ghaṭikās* before the opposition

$= 5$ signs $18° 25' 27''$;

∴ the moon's celestial latitude

$= 270' \times \sin 4° 9'' 27''$,

$= 19' 34''$

as in the preceding approximation.

The Indian astronomer would thus conclude that this eclipse would begin at 4·43 *ghaṭikās* or 4 *ghaṭikās* 25 *bināḍis* 48 *bipalas*, before the instant of opposition.

We now take up the calculation of the half duration for the end of the eclipse.

At 4·539033 *ghaṭikās* after the instant of opposition—

the longitude of the moon $= 5$ signs $20° 33' 25''$,

,, ,, ,, node $= 11$ signs $22° 34' 16''$,

∴ the moon's celestial latitude $= 270' \sin 2° 0' 51''$,

$= 9' 29''$.

The sum of the semi-diameters

$= 61' 59''$.

Hence the half duration for the end of the eclipse

$$= \frac{\sqrt{(16' 59'')^2 - (9' 29'')}}{856' 12'' - 59' 39''} \times 60 \text{ } ghaṭikās,$$

$= 4·613916$ *ghaṭikās*, which represents the *2nd approximation.*

Again at 4·613916 *ghaṭikās* after the instant of opposition,

the longitude of the moon $= 5$ signs $20° 34' 20''$,

and the longitude of the node $= 11$ signs $22° 34' 15''$.

The moon's celestial latitude $= 270' \sin 2°$,

$= 9' 25''$.

Hence the third approximation to the half duration for the end of the eclipse

$$= \frac{\sqrt{(61' 59'')^2 - (9' 25'')^2}}{856' 12'' - 59' 39''} \times 60 \text{ } ghaṭikās,$$

$= 4·6147$ *ghaṭikās*

$= 4$ *ghaṭikās* 36 *bināḍis* 53 *bipalas.*

As this agrees very well with the result of the last step, according to Hindu astronomers the time when the eclipse will end is to be taken at 4 *ghaṭikās* 36 *bināḍis* 53 *bipalas* after the instant of opposition.

We now take up the times of the beginning and of the end of the total obscuration.

Half the duration of the total obscuration has been approximately found to be 1·74303 *ghaṭikās*.

The longitude of the moon at 1·74303 *ghaṭikās* before the instant of opposition $= 5$ signs $19° 3' 38''$.

That of the node $= 11$ signs $22° 34' 36''$;

∴ the moon's celestial latitude at this time

$= 270' \sin 3° 30' 58''$,

$= 16' 32''$.

The difference of their semi-diameters

$= 27' 19''$.

Hence the half duration for the beginning of the total obscuration

$$= \frac{\sqrt{(27' 19'')^2 - (16' 32'')}}{856' 12'' - 59' 39''} \times 60 \text{ } ghaṭikās,$$

$= 1·6$[illegible] *ghaṭikās*, *which represents the second approximation.*

Again at 1·6[illegible] *ghaṭikās*, preceding the instant of opposition—

the longitude of the moon $= 5$ signs $19° 4' 57''$,

,, ,, node $= 11$ signs $22° 34' 35''$,

∴ the moon's celestial latitude $= 270' \sin 3° 29' 38''$,

$= 16' 29''$.

The difference of their semi-diameters

$= 27' 19''$.

Hence the third approximation to the half duration for the beginning of total obscuration

$$= \frac{\sqrt{(27'\ 19'')^2 - (16'\ 29'')^2}}{856'\ 12'' - 59'\ 39''} \times 60 \text{ ghaṭikās},$$

= 1·640803 *ghaṭikās*,

= 1 *ghaṭikā* 38 *bināḍīs* 27 *bipalas*.

Now the next step will yield the same result. Thus we are to take that the total obscuration will begin at 1 *ghaṭikā* 38 *bināḍīs* 27 *bipalas* before the instant of opposition.

We now pass on to find the time when the total obscuration will end. This has been roughly found to be 1·74803 *ghaṭikās* after the instant of opposition. The longitude of the moon at this time will be

= 5 signs 19° 55′ 22″.

The longitude of the node = 11 signs 22° 34′ 24″ ;

∴ the moon's celestial latitude = 270′ sin 2° 41′ 2″ = 12′ 39″.

The difference of the semi-diameters

= 27′ 19″.

Half duration for the end of total obscuration

$$= \frac{\sqrt{(27'\ 19'')^2 - (12'\ 39'')^2}}{856'\ 12'' - 59'\ 39''} \times 60 \text{ ghaṭikās},$$

= 1·8237 *ghaṭikās*.

Again the longitude of the moon at 1·8237 *ghaṭikās* after the instant of opposition

= 5 signs 19° 54′ 32″,

and that of the node = 11 signs 22° 34′ 24″ ;

∴ The moon's celestial latitude = 270′ sin 2° 39′ 52″,

= 12′ 36″.

The difference of the semi-diameters

= 27′ 19″.

Hence half duration for the end of the total obscuration

$$= \frac{\sqrt{(27'\ 19'')^2 - (12'\ 36'')^2}}{856'\ 12'' - 59'\ 39''} \times 60 \text{ ghaṭikās},$$

= 1·82566 *ghaṭikās*,

= 1 *ghaṭikā* 49 *bināḍīs* 32 *bipalas*.

As the result of the next step will also be the same we are to take that the total obscuration will end at 1 *ghaṭikā* 49 *bināḍīs* 32 *bipalas* after the instant of opposition. According to the direction of Pṛthūdaka, we put down the time of opposition in 5 places (here in 5 lines) and obtain the following instants:—

I. 5 *gh.* 5 *bin.* 2 *bip.* − 4 *gh.* 25 *bin.* 48 *bip.*,
= 0 *gh.* 39 *bin.* 14 *bip.*, after midnight for beginning of the eclipse.

II. 5 *gh.* 5 *bin.* 2 *bip.* − 1 *gh.* 38 *bin.* 27 *bip.*,
= 3 *gh.* 26 *bin.* 35 *bip.*, A.M. for the beginning of the total obscuration.

III. 5 *gh.* 5 *bin.* 2 *bip.* − 0 *gh.* 0 *bin.* 0 *bip.*,
= 5 *gh.* 5 *bin.* 2 *bip.*, A.M. for the instant of opposition.

IV. 5 *gh.* 5 *bin.* 2 *bip.* + 1 *gh.* 49 *bin.* 32 *bip.*,
= 6 *gh.* 54 *bin.* 34 *bip.*, A.M. for the end of total obscuration.

V. 5 *gh.* 5 *bin.* 2 *bip.* + 4 *gh.* 36 *bin.* 53 *bip.*,
= 9 *gh.* 41 *bin.* 55 *bip.*, A.M. for the end of the eclipse.

In the above calculation of the lunar eclipse we have followed the orthodox Hindu method; if we follow the *modern method*, the calculation with the old constants, will proceed as follows:—

The instant of opposition in Ujjayinī mean time is 3rd April, 1931, 5 *gh.* 5 *bin.* 2 *bip.* A.M. At this time—

The longitude of the moon = 5 signs 19° 28′ 30″.
That of the node = 11 signs 22° 34′ 30″.
The moon's celestial latitude = 14′ 31″ = 871″.
The moon's diameter = 34′ 40″.
The diameter of the shadow = 89′ 18″.
The sum of the semi-diameters
= 61′ 59″ = 3719″.
The moon's motion per *ghaṭikā* = 856″·2
The rate of change of moon's celestial latitude per *ghaṭikā*
= − 67″·33.
The sun's motion in longitude per *ghaṭikā*
= 59″·65.

Now δ, the distance between the centres of the moon and of the shadow after 't' *ghaṭikās* is given by the equation

$$\delta^2 = \{(856''\cdot 2 - 59''\cdot 65)t\}^2 + \{871'' - 67''\cdot 33 \times t\}^2$$

Now put (I) $\delta = 3719''$, the sum of the semi-diameters,

$\therefore \quad 3719^2 = (796{\cdot}55t)^2 + (871 - 67{\cdot}33t)^2$,

or $\quad 634945{\cdot}3334t^2 - 117288{\cdot}86t - 13072320 = 0$;

$\therefore \quad t_1 = -4{\cdot}446$ *ghaṭikās* $= -4$ *ghaṭikās* 27 *bināḍīs* which by the method of Indian astronomers worked out to be -4 *h.* 26 *bin.*

$t_2 = +4{\cdot}63$ *ghaṭikās* $= 4$ *ghaṭikās* 38 *bināḍīs* nearly, which by the Indian method came out to be 4 *ghaṭikās* 37 *bināḍīs* nearly.

Again (II) put $\delta = 27' \, 19'' = 1639''$, which is the difference of the semi-diameters.

The equation now becomes:

$$634945{\cdot}3334t^2 - 117288{\cdot}86t - 1927680 = 0,$$

$\therefore \; t_1 = -1{\cdot}652$ *ghaṭikās*, $= -1$ *ghaṭikā* 39 *bināḍīs* which by the Indian method came out to be -1 *ghaṭikā* 38 *bināḍīs*.

Also, $\quad t_2 = +1{\cdot}8373$ *ghaṭikās* $= 1$ *ghaṭikā* 50 *bināḍīs*, which by the Indian method worked out to be 1 *ghaṭikā* 49 *bināḍīs*.

Hence when the constants are the same, the Hindu method though rather tedious leads to almost the same results as the modern method.

The results of the calculation by the Indian method may be shown in a tabular form as follows:—

Phenomena	Ujjayinī local time (calculated)	Calcutta local time (calculated)	Greenwich mean time (calculated)	G. M. T. as in Conn. des Temps
Beginning of the Eclipse.	0 *h.* 16 *min.*	1 *h.* 6 *min.* 30 *sec.*	19 *hrs.* 12 *min.*	18 *hrs.* 23 *min.* 2 *sec.*
Beginning of the totality.	1 *h.* 23 *min.*	2 *hrs.* 13 *min.* 30 *sec.*	20 *hrs.* 19 *min.*	19 *hrs.* 22 *min.* 3 *sec.*
Instant of opposition.	2 *hrs.* 2 *min.*	2 *hrs.* 52 *min.* 30 *sec.*	20 *hrs.* 58 *min.*	20 *hrs.* 7 *min.* 4 *sec.*
End of totality.	2 *hrs.* 46 *min.*	3 *hrs.* 36 *min.* 30 *sec.*	21 *hrs.* 42 *min.*	20 *hrs.* 52 *min.* 6 *sec.*
End of the Eclipse.	3 *hrs.* 53 *min.*	4 *hrs.* 43 *min.* 30 *sec.*	22 *hrs.* 49 *min.*	21 *hrs.* 51 *min.* 7 *sec.*

The *Khaṇḍakhādyaka* constants thus bring in all the above phases of the eclipse by about 50 minutes later than the actual times.

5. The apparent daily motion whether of the sun or of the moon divided by 60 and multiplied by the half-durations of the eclipse or of the total obscuration must be repeatedly applied, negatively for the beginning and positively for the end. As to the *pāta*, or the node, the corrections are to be applied in the reverse order.

The motion of the node is retrograde, hence in its interpolation for a preceding time the correction is to be applied positively and for a subsequent time it is to be applied negatively. The processes described in this stanza have already been illustrated.

6. From the half duration of the eclipse, whether of the beginning or of the end, subtract the desired time after which or before which the phase is wanted ; by means of that time find the minutes of arc gained by the moon and also the moon's celestial latitude : by the square root of the sum of their squares lessen the sum of the semi-diameters, the result represents the obscured portion. In the middle of the eclipse the same is obtained by diminishing the sum of the semi-diameters by the moon's celestial latitude.

Illustration.—Suppose the phase is wanted at 1 *ghaṭikā* after the beginning of the eclipse. Here the half-duration of the eclipse as found in the previous calculation

= 4 *ghaṭikās* 25 *bināḍīs* 48 *bipalas*.

Now 4 *ghaṭikās* 25 *bināḍīs* 48 *bipalas* − 1 *ghaṭikā* = 3 *ghaṭikās* 25 *bināḍīs* 48 *bipalas*, represents the time preceding the instant of opposition, at which the phase is wanted.

The moon will be behind the centre of the shadow by

$$\frac{3 \text{ gh. } 25 \text{ bin. } 48 \text{ bip. } (856' \, 12'' - 59' \, 39'')}{60 \text{ gh.}} \text{ i.e., by } 45' \, 24'' \text{ ;}$$

The longitude of the moon at that time

$= 5$ signs $18° \, 39' \, 42''$.

That of the node $= 11$ signs $22° \, 34' \, 41''$;

$\therefore$ the moon's celestial latitude

$= 270' \sin 3° \, 55' \, 1''$,

$= 18' \, 26''$.

Now $\sqrt{(45' 24'')^2 + (18' 26'')^2} = 48' 54''$, which is the distance between the centres of the moon and the shadow.

The sum of the semi-diameters

$$= 61' 59'' ;$$

∴ the minutes of arc of the moon's disc. eclipsed

$$= 61' 59'' - 48' 54'' = 13' 5''.$$

By the converse of the above process the time for a required phase of the eclipse may be calculated.

7. Multiply the 'sine' of the hour angle by the 'sine' of the latitude and divide by the radius; the degrees of the arc of which this quotient is the 'sine,' are to be taken as of *north* and *south* denominations according as the obscured body lies on the eastern or the western half of the celestial sphere: by the sum or the difference of these degrees and the degrees of the declination of the obscured, increased by 3 signs or 90°, according as they are of the same or opposite denominations is obtained the variation of the eastward direction of the ecliptic from the eastward direction of the disc of the obscured body.

This stanza gives the rule for calculating the *Valana*, or the variation of the eastward direction of the ecliptic from the eastward direction of the obscured body. The rule itself is identical with that of the *Āryabhaṭīya*, *Gola*, 45, *B.S. Siddhānta*, IV, 16-18, *Sūrya-Siddhānta*, IV, 24-25. Most accurate rules were first given by Bhāskara II, *Gola*, VIII, 30-74, and the commentary thereon. See also Papers on Hindu Mathematics and Astronomy, *Valana*, by the translator.

The rough rule given here is being illustrated. At the instant of opposition, at 5 *ghaṭikās* 5 *bināḍis* 2 *bipalas* of mean time the hour angle of the mean sun equals 24 *ghaṭikās* 54 *bināḍis* 58 *bipalas*.

Now the apparent sun is ahead of the mean sun on the ecliptic by the equation of the centre, *viz*., 2° 13′ 50″, *i.e.*, by 22 *bin*. 18 *bip*. Hence the apparent sun's hour-angle = 24 *gh*. 54 *bin*. 58 *bip*. + 22 *bin*. 18 *bip*.

Therefore the moon's hour angle

= 5 *gh*. 5 *bin*. 2 *bip*. − 22 *bin*. 18 *bip*.

= 4 *gh*. 42 *bin*. 44 *bip*.

= 28° 16′ 24″ West at Ujjayinī.

At Calcutta this will be

= 28° 16′ 24″ + 12° 38′

= 40° 54′ 24″ West.

Now the moon's longitude at the instant of conjunction from the vernal equinoctial point

= total precession at 1853 *Śaka* year + 5 signs 19° 28′ 30″

= 23° 21′ 20″ + 5 signs 19° 28′ 30″

= 6 signs 12° 49′ 57″.

Therefore, moon + 90° = 9 signs 12° 49′ 57″

Hence declination of (moon + 90°)

$= -\sin^{-1} (\sin 24° \times \sin 77° 10' 3'')$

= 23° 22′ South nearly.

Again at Calcutta the latitude is 22° 35′ ;
hence the first part of *Valana*

$= \sin^{-1} (\sin 40° 54' 24'' \times \sin 22° 35')$

= 14° 34′ South ;

∴ the total variation of the east of the ecliptic

= 14° 34′ + 23° 22′ = 37° 56′ south from the east of the disc of the moon.

Note.—Āryabhaṭa, Varāhamihira, Brahmagupta and all Indian astronomers before the time of Śrīpati (1028 A.D.), could not discover the part of the equation of time due to the obliquity of the ecliptic, or that the uniform measure of the *ahargaṇa* could only be got by the mean sun moving uniformly with the sun's mean rate not along the ecliptic but along the equator.

Pṛthūdaka ends his commentary on this chapter by promising to explain what to do with this *Valana*, in the projection of eclipses in his exposition of the supplementary part of the *Khaṇḍakhādyaka*. But the manuscripts at our disposal give only the beginning of this supplementary part and it is not possible to say if he fulfilled his promise. As to the projection of the eclipses see *Sūrya-siddhānta*, VI, Burgess' Translation.

This brings us to the end of Chapter IV of the Khaṇḍakhādyaka, which relates to the Lunar Eclipses.

CHAPTER V

On Solar Eclipses.

As this chapter treats of the eclipses of the sun we proceed, according to the *Khaṇḍakhādyaka* constants, with the calculation of the solar eclipse on the 9th May, 1929, or the Śaka era 1851, one synodic month; the day of the week being Thursday. The station is Calcutta, the longitude of which is 88° 30′ E. and the latitude 22° 35′ N.

Here the *ahargaṇa* = 461711 at the mean midnight at Ujjayinī (Ojein; long. 75° 52′ E., and lat. 23° 11′ N.) on this Thursday.

The mean sun	= 0 sign 24° 8′ 6″.
The mean moon	= 1 sign 5° 59′ 46″.
The sun's apogee	= 2 signs 20°.
The moon's apogee	= 8 signs 17° 51′ 35″.

Now, mean moon − mean sun = 11° 51′ 40″; hence the true instant of conjunction cannot be calculated by using the apparent positions at midnight. We take the mean longitudes for the preceding mean midday at Ujjayinī, which were:—

The mean sun	= 0 sign 23° 38′ 32″.
The mean moon	= 0 sign 29° 24′ 28″.
The sun's apogee	= 2 signs 20°.
The moon's apogee	= 8 signs 17° 48′ 15″.
Now, the sun's equation	= + 134′ sin 56° 21′ 27″,
	= + 1° 51′ 34″;
∴ the apparent sun	= 0 sign 25° 30′ 6″.
The moon's equation	= − 296′ sin 48° 23′ 47″,
	= − 3° 41′ 20″;
∴ the apparent moon	= 0 sign 25° 43′ 8″.

Thus the instant of conjunction was already over at the mean midday, the moon having gained 13′ 2″ over the sun.

Now, the moon's apparent instantaneous daily motion

$$=790'\,35''+\frac{783'\,54''\times 147\times 31}{214\times 360}=836'\,57''.$$

The sun's apparent instantaneous daily motion

$$=59'\,8''-\frac{59'\,8''\times 113\times 14}{214\times 360}=57'\,55''.$$

The difference of their daily motions
= 779′ 12″ = 46742″.

Hence the instant of conjunction

$$=\text{Mean moon at Ujjayinī}-\frac{13'\,2''\times 60}{46742''}\ ghaṭikās.$$

= Ujjayinī mean moon − 1·00381 *ghaṭikās*.
= 11 *hrs.* 35 *min.* 54 *secs.* A.M. of Ujjayinī mean time.
= 12 *hrs.* 26 *min.* 26 *secs.* A.M. of Calcutta mean time.
= **6 *hrs.* 32 *min.* 26 *secs.* A.M. of G. M. T.**

The true instant of conjunction as given in the Conn. des Temps, 1929, page 52 is **6 *hrs.* 7 *min.*** G. M. T. Thus there is an error of 25 *min.* in the calculated instant of conjunction.

The sun's longitude at the calculated instant
= 0 sign 25° 29′ 8″, which is also the longitude of the moon.

The longitude of the node at the same instant with Pṛthūdaka and Lalla's correction
= 0 sign 29° 19′ 30″.

The total shifting of the equinoxes from 421 to 1851 of the Śaka era and one synodic month
= 23° 16′ 7″.

Hence the longitude of the sun from the true equinox of date
= 48° 45′ 15″.

The sun's declination accordingly
$= \sin^{-1}(\sin 24° \times \sin 48°\,45'\,15'')$
= 17° 48′ 27″.

The hour angle of the mean sun (*i.e.*, the mean sun on the ecliptic according to the early Hindu astronomers) at the instant of conjunction = 0 *hr.* 26 *min.* 26 *secs.* W.
= 6° 36′ 30″ West.
sun's equation = 1° 51′ 34″;

hence the apparent sun's hour angle	= 4° 44′ 56″ W.
	= 0 *gh.* 47 *bin.* 29·33 *bip.*
The latitude of Calcutta	= 22° 35′ N.
The sun's declination	= 17° 48′ 27″ N.;

∴ the length of half the day at Calcutta
= 16 *gh.* 16 *bin.* 46 *bip.*

The time elapsed since sunrise = 17 *gh.* 4 *bin.* 15·33 *bip.*

It is now necessary to find the long. de of the orient ecliptic point and we need to determine the time du tions for the risings of the different signs of the zodiac at Calcutta, which are worked out below:—

Signs	Durations in *asus* for the risings on Equator	Tab. difference in *asus* of ascensional differences at Calcutta	Durations in *asus* for the risings of signs at Calcutta	Signs
Aries	1669	−297	1372	Pisces
Taurus	1794	−243	1551	Aquaris
Gemini	1937	−100	1837	Capricorn
Cancer	1937	+100	2037	Sagittarius
Leo	1794	+243	2037	Scorpio
Virgo	1669	+297	1966	Libra

The sun's longitude = 48° 45′ 15″
= 1 sign 18° 45′ 15″ ;

∴ the residue of two signs = 11° 14′ 45″ ;

this rises in $\frac{11° 14' 45'' \times 1551}{30°}$ *asus*,

= 581·41 *asus* ;

Gemini rises in 1837 *asus* ;

Cancer rises in 2037 *asus*.

Total = 4455·41 *asus*.

Time elapsed since sunrise = 17 *gh.* 4 *bin.* 29·33 *bipalas*.
= 6145·53 *asus*.

Time elapsed of the rising of Leo
= 1690·12 *asus*,

which corresponds to $\frac{1690{\cdot}12 \times 30°}{2037}$

= 24° 53′ 29″ secs. of Leo.,

∴ the longitude of the orient ecliptic point
= 144° 53′ 29″.

The longitude of the nonagesimal
= 54° 53′ 29″.

The declination of the nonagesimal
= 19° 26′ 5″.

Again, moon−node, at the instant of conjunction
= 11 signs 26° 9′ 38″ ;

∴ the moon's celestial latitude = −270′ sin 3° 50′ 22″
= −18′ 4″.

The sun's diameter $= \frac{57' 55'' \times 11}{20}$,
= 31′ 51·25″.

The moon's diameter $= \frac{836' 57'' \times 10}{247}$,
= 33′ 53″·08.

It is now necessary to collect all the elements found, before we proceed any further.

(1) The instant of conjunction in Cal. M. T.
= 12 *hrs.* 26 *min.* 26″.

(2) The sun's longitude at conjunction
= 0 sign 25° 29′ 8″.

(3) The total shifting of the equinoctial point from 421 of Saka era (*i.e.*, 499 A.D.) till date of eclipse
= 23° 16′ 7″ according to the *Khaṇḍakhādyaka* year.

(4) Sun's longitude from true equinox
= 48° 45′ 15″.

(5) Sun's declination = 17° 48′ 27″.

(6) Sun's hour-angle = 0 *gh.* 47 *bin.* 29·33 *bip.*

(7) Length of the day at Calcutta
= 16 *gh.* 16 *bin.* 46 *bip.*

(8) Time elapsed since sunrise = 17 *gh.* 4 *bin.* 15·33 *bip.*

(9) Longitude of the orient ecliptic point
= 144° 53′ 29″.

(10) Longitude of the nonagesimal
= 54° 53′ 29″.

(11) Declination of the nonagesimal
= 19° 26′ 5″.

(12)	The node's longitude	= 0 sign 29° 19′ 30″.
(13)	The moon's celestial latitude	= −18′ 4″.
(14)	The sun's apparent daily motion	= 57′ 55″.
(15)	The moon's apparent daily motion	= 836′ 57″.
(16)	The sun's apparent diameter	= 31′ 51·25″.
(17)	The moon's ,, ,,	= 33′ 53·08″.
(18)	The latitude of the station	= 22° 35′ N.

We are now in a position to consider the first two stanzas of this chapter, which relate to the parallactic shifting of the instant of conjunction.

1-2. From three signs (*i.e.*, 90°) deduct the sum or difference (*i.e.*, algebraic sum) of the declination of the nonagesimal, the moon's celestial latitude and the latitude of the station (which is always taken,— in the northern hemisphere); the 'sine' of the remainder is the divisor of the square of half the radius ; by this quotient divide the 'sine' of the arc between the nonagesimal and the sun ; apply the result, after repeated operations, taken as *ghaṭikās* which represents the parallactic shifting—to the instant of conjunction, negatively when the sun is greater than the nonagesimal; and positively when the sun is less than the nonagesimal.

The idea is to find the apparent instant of conjunction at the place of the observer. At the instant of conjunction the sun, the moon (with no celestial latitude) and the centre of the earth are in the same straight line. At the place of the observer, the moon is depressed from the straight line joining the observer and the sun, by the angle which is equal to the difference between the parallaxes of the sun and of the moon. The projection of this depression on the ecliptic measures the shifting of the moon from the sun in longitude, and its projection perpendicular to the ecliptic, the apparent change in the celestial latitude. If this shifting in longitude is ahead of the sun, the apparent instant of conjunction is over; if it be behind the sun the conjunction is to come. The above stanzas teach us how to find this time by which the apparent conjunction is over or is to come.

All Indian astronomers from the time of Brahmagupta take the horizontal parallax of a planet $= \frac{1}{15}$ of its daily motion in longitude. If n' and n be the apparent daily motions of the moon and the sun, then the depression of the moon from the sun $= \frac{1}{15}(n'-n)$ in the vertical circle, if at the instant of conjunction both the sun and the moon be on the horizon. If in addition the ecliptic be vertical then $\frac{1}{15}(n'-n)$ is the deflection in longitude of the moon from the sun, which is made up by the moon in $\frac{\frac{1}{15}(n'-n) \times 60}{(n'-n)}$ or 4 *ghaṭikās*. It is this 4 *ghaṭikās*, which occurs in the equation for the apparent instant of conjunction at the place of the observer.

Let $\odot$ stand for the sun's longitude, N for that of the nonagesimal, Z' the zenith distance of the nonagesimal; then the import of the stanzas may be expressed symbolically thus:—

Z' = declination of the nonagesimal + latitude of the moon + latitude of the station, the latitude of the station being always called south (or −) in the northern hemisphere.

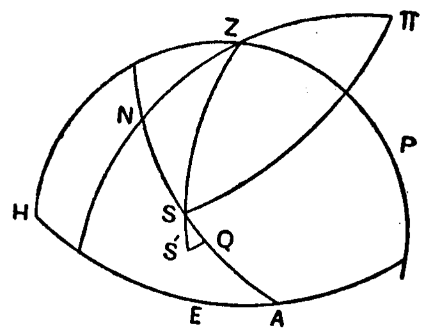
The equation of apparent conjunction

$$= -\frac{\frac{R \sin(\odot - N)}{R^2}}{4R \sin(90° - Z')} \text{ } ghaṭikās$$

$$= -\frac{4 R \cos Z' \times R \sin(\odot - N)}{R^2} \text{ } ghaṭikās.$$

The truth of this equation is seen thus:—

In the following figure, let PZH be the observer's meridian, HEA the horizon, AQN the ecliptic, N the nonagesimal, SS' the deflection of the moon from the sun; then $SS' = (\pi_m - \pi_s) \times \frac{R \sin ZS}{R}$, where π_m and π_s denote the horizontal parallaxes of the moon and the sun respectively, Z the zenith of the observer. From S' and Z draw $S'Q$ and ZN perpendicular to the ecliptic. Let the secondaries to the ecliptic at N and S meet at its pole Π.



Now $SQ = SS' \times \frac{R \cos S'SQ}{R}$

$$= (\pi_m - \pi_s) \times \frac{R \sin ZS}{R} \times \frac{R \cos S'SQ}{R}.$$

Again $\sin NS = \tan ZN \times \cot S'SQ$,

also $\sin ZN = \sin ZS \times \sin S'SQ$,

$\therefore \sin NS \times \cos ZN = \sin ZS \cos S'SQ$.

$$\therefore SQ = (\pi_m - \pi_s) \times \frac{R \sin NS \times R \cos ZN}{R^2}.$$

This deflection SQ is made up by the moon in

$$4 \times \frac{R \sin NS \times R \cos ZN}{R^2} \text{ ghaṭikās.}$$

$$\text{or} \quad 4 \times \frac{R \sin (\odot - N) \times R \cos Z'}{R^2} \text{ ghaṭikās.} \qquad (1)$$

Again $S'Q$, the deflection in latitude

$$= \tfrac{1}{15}(n' - n) \times \frac{R \sin ZS}{R} \times \frac{R \sin S'SQ}{R}$$

$$= \tfrac{1}{15}(n' - n) \times \frac{R \sin ZN}{R}$$

$$= \tfrac{1}{15}(n' - n) \times \frac{R \sin S'}{R} \qquad (2$$

For the rigid Indian method of working out the above equations, the reader is referred to the translator's paper "Parallax in Hindu Astronomy" published in the Report of the Indian Association for the Cultivation of Science, Calcutta, for the year 1916, page 15.

Illustration.—In the calculation of the proposed solar eclipse at Calcutta on the 9th May, 1929, we have found that the declination of the nonagesimal = 19° 26′ 5″, the latitude of the station = −22° 35′, and the moon's celestial latitude = −18′ 4″.

$\therefore Z' = -3° 26' 59''$, $\odot - N = 6° 8' 14''$;

$\therefore$ the equation of apparent conjunction

= 4 cos 3° 26′ 59″ × sin 6° 8′ 14″

= ·426863 *ghaṭikās*.

= 153·67 *asus*.

(*ii*) Again the sun's motion in ·426863 *ghaṭikās*

	= 25″.
The sun's longitude from true equinox	= 48° 45′ 40″.
Time elapsed of the day	= 6229·20 *asus*.
Longitude of the orient ecliptic point	= 147° 9′ 35″
,, ,, ,, nonagesimal	= 57° 9′ 35″.
Declination of the ,,	= 19° 58′ 57″.
Moon's motion in ·426863 *ghaṭikās*	= 0° 5′ 51″.
Moon's long. at ·426863 *gh.* after conj.	= 0 sign 25° 35′ 5″.
Longitude of the node	= 0 sign 29° 19′ 29″.
Moon's celestial latitude	= 270′ sin 3° 44′ 24″ = −17′ 37″.
$Z' = -2° 53' 40''$; $\odot - N$	= −8° 23′ 55″.
The equation of apparent conjunction	= 4 cos 2° 53′ 40″ × sin 8° 23′ 55″.
	= ·5835 *ghaṭikās*
	or 210·06 *asus*,

which represents the second approximation.

(*iii*) Again at ·5835 *ghaṭikās* after the instant of conjunction:—

The sun's longitude	= 0 sign 25° 29′ 42″.
,, moon's ,,	= 0 sign 25° 37′ 15″.
,, node's ,,	= 0 sign 29° 19′ 28″.
The moon's celestial latitude	= −270′ sin 3° 42′ 13″,
	= −17′ 26″.
Time elapsed since sunrise	= 6355·59 *asus*.
Longitude of the orient ecliptic point	= 148° 7′ 49″.
,, ,, ,, nonagesimal	= 58° 7′ 49″.
Declination of the ,,	= 20° 12′ 26″
$Z' = -2° 40''$; $\odot - N$	= −9° 22′.
The equation of apparent conjunction	= 4 cos 2° 40′ × sin 9° 22′
	= ·650303 *ghaṭikās*.
	= 234·11 *asus*, which

represents the third approximation.

(*iv*) Again at ·650303 *ghaṭikās* after the instant of conjunction:—

The sun's longitude	= 0 sign 25° 29′ 46″.
,, moon's ,,	= 0 sign 25° 38′ 16″.
,, node's ,,	= 0 sign 29° 19′ 28″.
,, moon's celestial latitude	= −270′ sin 3° 41′ 12″.
	= −17′ 21″.

The sun's longitude from true equinox = 48° 45′ 53″.
Time elapsed since sunrise = 6379·64 *asus*.
The long. of the orient ecliptic point = 148° 20′ 50″.
The long. of the nonagesimal = 58° 20′ 50″.
The declination of the nonagesimal = 20° 15′ 25″.
$Z' = -2° 36' 56''$; $\odot - N = -9° 34' 57''$.
The equation of apparent conjunction = 4 cos 2° 36′ 56″ × sin 9° 34′ 57″
= ·665178 *ghaṭikās*,
= 239·46 *āsus*,

which represents the 4th approximation.

(*v*) Again at ·665178 *ghaṭikās* after the instant of conjunction:—

The sun's longitude = 0 sign 25° 29′ 47″.
The moon's ,, = 0 sign 25° 38′ 25″.
The node's ,, = 0 sign 29° 10′ 28″.
The moon's celestial latitude = −17′ 21″.
Time elapsed since sunrise = 6384·99 *asus*.
Sun's long. from true equinox = 48° 45′ 54″.
Long. of the orient ecliptic point = 148° 33′ 59″.
,, ,, ,, nonagesimal = 58° 33′ 39″.
Declination of the nonagesimal = 20° 18′ 24″.
$Z' = -2° 33' 57''$; $\odot - N = -9° 48' 5''$.

The equation of apparent conjunction = ·6802508 *ghaṭikās*, which represents the 5th approximation.

(*vi*) Again at ·6802508 *ghaṭikās* after the instant of conjunction:—

The sun's longitude = 0 sign 25° 29′ 47″.
,, moon's ,, = 0 sign 25° 38′ 37″.
,, node's ,, = 0 sign 29° 19′ 28″.
,, moon's celestial latitude = −17′ 20″.
Time elapsed since sunrise = 6390·42 *asus*.
The sun's longitude from true equinox = 48° 45′ 54″.
Longitude of the orient ecliptic point = 148° 30′ 20″.
,, ,, ,, nonagesimal = 58° 30′ 20″.
Declination of the ,, = 20° 17′ 33″.
$Z' = -2° 34' 49''$; $\odot - N = -9° 44' 26''$.
The equation of apparent conjunction = ·676063 *ghaṭikās*,
= 243·38 *asus*,

which represents the sixth approximation.

(*vii*) Again at ·676063 *ghaṭikās* after the instant of conjunction:—

The sun's longitude = 0 sign 25° 29′ 47″.
,, moon's ,, = 0 sign 25° 38′ 34″.
,, node's ,, = 0 sign 29° 19′ 28″.
,, moon's celestial latitude = −17′ 20″.
Time elapsed since sunrise = 6388·91 *asus*.
The sun's long. from true equinox = 48° 45′ 54″.
The long. of the orient ecliptic point = 148° 29′ 3″.
The longitude of the nonagesimal = 58° 29′ 3″.
The declination of the ,, = 20° 17′ 17″.
$Z' = -2° 35' 3''$; $\odot - N = -9° 43' 9'$.
The equation of apparent conjunction = ·674597 *ghaṭikās*
= 242·85 *asus*,

which represents the seventh approximation.

(*viii*) Again at ·674597 *ghaṭikās* after the instant of conjunction:—

The longitude of the sun = 0 sign 25° 29′ 47″.
,, ,, ,, ,, moon = 0 sign 25° 38′ 33″.
,, ,, ,, ,, node = 0 sign 29° 19′ 28″.
The moon's celestial latitude = −17′ 20″.
Time elapsed since sunrise = 6388·38 *asus*.
Sun's long. from true equinox = 48° 45′ 54″.
Long. of the orient ecliptic point = 148° 29′ 28″.
,, ,, ,, nonagesimal = 58° 29′ 28″.
Declination of ,, = 20° 17′ 22″.
$Z' = -2° 34' 58''$; $\odot - N = -9° 43' 34''$.
The equation of apparent conjunction = ·675068 *ghaṭikās*,
= 243·04 *asus*,

which represents the eighth approximation.

As the result of this approximation is almost the same as the previous one, the Indian astronomer would now take that the apparent instant of conjunction is ·675068 *ghaṭikās* or 243·04 *asus* after the geocentric conjunction. At this time of apparent conjunction:—

The longitude of the sun = 0 sign 25° 29′ 47″.
,, ,, ,, ,, moon = 0 sign 25° 38′ 33″.
,, ,, ,, ,, node = 0 sign 29° 19′ 28″.
The moon's celestial latitude = −17′ 20″.

Sun's long. from true equinox = 48° 45′ 54″.
Time elapsed since sunrise = 6388·58 *asus*.
Long. of the orient ecliptic point = 148° 29′ 21″.
,, ,, ,, nonagesimal = 58° 29′ 21″.
Declination of the nonagesimal = 20° 17′ 21″.
$Z' = -2° 34' 59''$; $\odot - N = -9° 43' 27''$.

Thus in Calcutta mean time, the instant of apparent conjunction (which is taken as the middle of the eclipse) is at 12 *hrs.* 42 *min.* 38 *secs.*

3-4. The 'sine' of the degrees of the sum or difference (*i.e.*, Z') multiplied by 13 and divided by 40 is the *avanati* or parallax in latitude; find the celestial latitude from the moon for the instant of apparent conjunction: the sum or difference according as they are of the same or different directions, of the parallax in latitude and the moon's celestial latitude, is the apparent celestial latitude. Then find the half durations in *ghaṭikās*, both of the eclipse and of the totality as in the case of a lunar eclipse.

The parallax in latitude is here given as

$$= \frac{13}{40} \times 150 \sin Z', \text{ and has been proved to be}$$

$$= \frac{n'-n}{15} \times \frac{R \sin Z'}{R}.$$

Now the average value of $n' = 790'\ 35''$, and of $n = 59'\ 8''$, and $R = 150$.

$$\therefore \frac{n'-n}{15} \times \frac{1}{R} = \frac{731'\cdot15}{15 \times 50} = \frac{14623'}{45000}$$

$$= \frac{1}{3+}\ \frac{1}{12+}\ \frac{1}{1+}\ \frac{1}{13+}\ \frac{1}{7+}\ \ldots\ldots\ldots\ldots;$$

Brahmagupta takes here the third convergent, *viz.*,

$$\frac{13'}{40}.$$

Illustration.—In this particular solar eclipse at Calcutta, we have found before that—

$$Z' = -2° 34' 59'' ;$$

$$\therefore \text{ parallax in latitude} = -\frac{13}{40} \times 150 \sin 2° 34' 39''$$

$$= -2' 11''.$$

The moon's celestial latitude $= -17' 20''$;

∴ the moon's apparent celestial latitude at the instant of apparent conjunction $= -19' 31''$.

Now the sun's diameter = 31′ 51″,
the moon's diameter = 33′ 53″;
∴ the sum of the semidiameters = 32′ 52″.
The portion of the sun obscured = 13′ 21″
(∴ the magnitude of the eclipse according to Hindu astronomers = ·419).

Hence the half duration of the eclipse

$$= \frac{\sqrt{(32' 52'')^2 - (19' 31'')^2}}{836' 57'' - 57' 55''} \times 60\ gh.$$

$$= 2\cdot03663\ ghaṭikās,$$

which represents the first approximation.

Now taking the moon's parallax in latitude to be constant for the entire duration of the eclipse, we are to carry on the successive approximations to (*a*) the half duration for the beginning and also to (*b*) the half durations for the end of the eclipse. The processes of successive approximations are here contracted by the modern process as shown in the previous chapter thus:—

Here the rate of increase of the moon's celestial latitude,
= 65″·185 per *ghaṭikā*.

The rate of increase of the sun's longitude,
= 57″·92 per *ghaṭikā*.

The rate of increase of the moon's longitude,
= 836″·95 per *ghaṭikā*.

Let *t* *ghaṭikās* be the time in which the distance between the centres of the sun and the moon becomes equal to the sum of their semidiameters, which is here 32′ 52″.;

$\therefore (32' 52'')^2 = (779''\cdot 03 \times t)^2 + (-19' 31'' + 65'' 19t)^2,$

$$\therefore t = \frac{152674\cdot 98 \pm \sqrt{(152674\cdot 98)^2 + 4 \times 611187\cdot 477 \times 2517543}}{2 \times 611187\cdot 477},$$

$= 2\cdot 15674$ *gh.* or $-1\cdot 90881$ *gh.*, the first of which is the half duration for the end of the eclipse and the second, the half duration for the beginning of the eclipse.

5-6. Again from the time of the middle of the eclipse, increased or decreased by the half duration of the eclipse or total obscuration, find by 'repeated process' as before the parallax in longitude in time; increase the corresponding half duration by its excess over the parallax in longitude for the middle; the result will be the corrected half duration. Similarly is found a half duration of total obscuration. As when the sun is greater than the nonagesimal, the sine of the arc between (divided by the *cheda* or divisor) is the parallax in longitude, its denomination is then negative, so it is positive in the reverse case. In this way when the parallaxes for the beginning and the middle are both positive or both negative, and that for the beginning is less, the half duration is decreased; if of different signs the same is increased by their sum. Similar is the rule for the end of the eclipse or of total obscuration.

The rule does not appear to be sufficiently clear. Let C denote the time for the instant of geocentric conjunction, and π_1, π_2 and π_3 denote the parallaxes in longitude, expressed in time for the beginning, middle and the end of the eclipse and let D_1 and D_2 be the half durations for the beginning and the end of the eclipse, then—

Time for the beginning of the eclipse	=	$C - D_1 \pm \pi_1$
Time for the middle of the eclipse	=	$C \pm \pi_2$
,, ,, ,, end ,, ,, ,,	=	$C + D_2 \pm \pi_3$,
∴ apparent half duration for the beginning of the eclipse	=	$D_1 \pm \pi_2 \mp \pi_1$;
apparent half duration for the end of the eclipse	=	$D_2 \pm \pi_3 \mp \pi_2$.

(*a*) Hence if π_1, π_2 and π_3 are all positive, D_1 is increased and D_2 is decreased if $\pi_1 < \pi_2$, and $\pi_3 < \pi_2$.

(*b*) If π_1, π_2 and π_3 are all $-$, D_1 is decreased and D_2 is increased.

(*c*) If π_2 is $+$, and π_1 is $-$, π_3 is $+$, D_1 becomes $D_1 + \pi_2 + \pi_1$ and D_2 becomes $D_2 + \pi_3 + \pi_2$, etc., etc.

All these rules cannot be combined in any one single rule.

Illustration.—In this particular eclipse the Calcutta mean time for the middle of the eclipse has been found before to be—

12 *hrs.* 42 *min.* 38 *secs.*

or, 6388·58 *asus* elapsed since sunrise.

Half duration for the beginning of the eclipse,

= 1·90881 *ghaṭikās*.

= 687·17 *asus*.

(*i*) Now at 1·90881 *ghaṭikās* before the instant of apparent conjunction—

The sun's longitude	= 0 sign 25° 27′ 56″.
The moon's longitude	= 0 sign 25° 11′ 55″.
The node's longitude	= 0 sign 25° 19′ 34″.
The moon's celestial latitude	= −19′ 24″.
Longitude of sun from true equinox	= 48° 44′ 3″.
Time elapsed since sunrise	= 5701·41 *asus*.
Longitude of the orient ecliptic point	= 138° 20′ 2″.
Longitude of the nonagesimal	= 48° 20′ 2″.
Declination of the nonagesimal	= 17° 41′ 19″.
$Z' = -5° 13' 5''$;	$\odot - N = 24' 1''$.
∴ the corresponding parallax	$= -4 \cos 5° 13' 5'' \times \sin 23' 55''$ *gh.*
	= −·02771 *ghaṭikās*.

Now parallax in longitude for the middle of the eclipse,

= +·675068 *ghaṭikās*;

∴ the corrected half duration for the beginning of the eclipse

= 2·61159 *ghaṭikās*.

= 820·17 *asus*.

(*ii*) Again at 2·61159 *ghaṭikās* before the instant of apparent conjunction—

The sun's longitude	= 0 sign 25° 27′ 16″.
The moon's longitude	= 0 sign 25° 2′ 7″.

The node's longitude = 0 sign 29° 19′ 36″.

The moon's celestial latitude = −20′ 12″.

Long. of the sun from true equinox = 48° 43′ 23″.

Time elapsed since sunrise = 5568·41 *asus*.

Long. of the orient ecliptic point = 136° 22″ 5″

Long. of the nonagesimal = 46° 22′ 5″.

Declination of the nonagesimal = 17° 7′ 16″.

$Z' = -5° 47' 56''$; $\odot - N = 2° 21' 18''$;

∴ the corresponding parallax = − ·16356 *ghaṭikās*;

∴ the next approximation to the half duration for the beginning of the eclipse, = 2·7474 *ghṭikās*.

= 989·08 *asus*.

(*iii*) Again at 2·7474 *ghaṭikās* before the instant of apparent conjunction—

The longitude of the sun = 0 sign 25° 27′ 8″.

The longitude of the moon = 0 sign 25° 0′ 14″.

The longitude of the node = 0 sign 29° 19′ 37″.

The moon's celestial latitude = −20′ 21″.

Long. of the sun from true equinox = 48° 43′ 15″.

Time elapsed since sunrise = 5399·50 *asus*.

Long. of the orient ecliptic point = 133° 52′ 36″.

Long. of the nonagesimal = 43° 52′ 36″.

Declination of the nonagesimal = 16° 22′ 28″.

$Z' = -6° 32' 53''$; $\odot - N = 4° 50' 39''$;

∴ the corresponding parallax = − ·33558 *ghaṭikās*.

The parallax for the middle of the eclipse

= ·675068 *ghaṭikās*.

∴ the next approximation to the half duration for the beginning of the eclipse = 2·91946 *ghaṭikās*.

= 1051 *asus*.

(*iv*) Again at 2·91946 *ghaṭikās* before the instant of apparent conjunction—

The longitude of the sun = 0 sign 25° 26′ 58″.

The longitude of the moon = 0 sign 24° 57′ 50″.

The longitude of the node = 0 sign 29° 19′ 37″.

The moon's celestial latitude = −20′ 32″.

Long. of the sun from true equinox = 48° 43′ 5″.

Time elapsed since sunrise = 5337·58 *asus*.

Long. of the orient ecliptic point = 132° 57′ 53″.

Long. of the nonagesimal = 42° 57′ 53″.

Declination of the nonagesimal = 16° 5′ 27″

$Z' = -6° 50' 5''$ $\odot - N = 5° 45' 12''$.

The corresponding parallax = − ·39815 *ghaṭikās*;

∴ the next approximation to the half duration for the beginning of the eclipse = 2·98203 *ghaṭikās*.

= 1073·53 *asus*.

(*v*) Again at 2·98203 *ghaṭikās* before the instant of apparent conjunction—

The longitude of the sun = 0 sign 25° 26′ 54″.

The longitude of the moon = 0 sign 24° 56′ 57″.

The longitude of the node = 0 sign 29° 19′ 37″.

The moon's celestial latitude = −20′ 37″.

Long. of the sun from true equinox = 48° 43′ 1″.

Time elapsed since sunrise = 5315·05 *asus*.

Long. of the orient ecliptic point = 132° 37′ 1″.

Long. of the nonagesimal = 42° 37′ 1″.

Declination of the nonagesimal = 15° 59′ 9″.

$Z' = -6° 56' 28''$; $\odot - N = 6° 6' 0''$.

The corresponding parallax in long. = ·39566 *ghaṭikās*.

The parallax for the middle of the eclipse

= ·67507 *gh*.

Mean half duration for the beginning of the eclipse

= 1·90881 *gh*.

The next approximation to the half duration for the beginning of the eclipse = 2·97954 *ghaṭikās*.

As this result is almost the same as of the previous step, the Hindu astronomers would finally take 2·98 *ghaṭikās* or 71 *min.* to be the correct half duration for the beginning of this solar eclipse.

Now the instant of apparent middle of the eclipse

= 12 *hrs*. 43 *min*. Cal. M. T.

Apparent half duration for the beginning of the eclipse

= 1 *hr*. 11 *min* ;

∴ the instant of the beginning of the eclipse

= 11 *hrs*. 32 *min*. of Calcutta Mean Time.

We now proceed to determine the half duration for the end of this solar eclipse; this has been approximately found to be 2·15674 *ghaṭikās* = 776·43 *asus*.

(*i*) Now at 2·15674 *ghaṭikās* after the instant of apparent conjunction—

The longitude of the sun = 0 sign 25° 31′ 51″.
The longitude of the moon = 0 sign 26° 8′ 38″.
The longitude of the node = 0 sign 29° 19′ 21″.
The moon's celestial latitude = −14′ 58″.
Longitude of the sun from the true equinox
= 48° 47′ 58″.
Time elapsed since sunrise = 7165·01 *asus*.
Long. of the orient ecliptic point = 160° 17′ 58″.
Long. of the nonagesimal = 70° 17′ 58″.
Declination of the nonagesimal = 22° 30′ 55″.

$Z' = -19' 8''$. ; $\odot - N = -21° 30'$.

The corresponding parallax in long. = +1·4326 *ghaṭikās*.

Parallax in long. for the middle of the eclipse
= +·67517 ,,

Mean half duration for the end of the eclipse
= 2·15674 *gh*.

The first approximation to the half duration for the end of the eclipse = 2·94417 *gh*.
= 1059·90 *asus*.

(*ii*) Again at 2·94417 *ghaṭikās* after the instant of apparent conjunction—

The longitude of the sun = 0 sign 25° 32′ 38″.
The longitude of the moon = 0 sign 26° 19′ 37″.
The longitude of the node = 0 sign 29° 19′ 19″.
The moon's celestial latitude = −14′ 8″.
Long. of the sun from true equinox = 48° 48′ 45″.
Time elapsed since sunrise = 7448·48 *asus*.
Long. of the orient ecliptic point = 164° 38′ 6″.
Long. of the nonagesimal = 74° 38′ 6″.
Declination of the nonagesimal = 23° 5′ 29″.

$Z' = +16' 21''$; $\odot - N = -25° 49' 21''$.

The corresponding parallax in long. = 1·74232 *ghaṭikās*.

The second approximation to the half duration for the end of the eclipse = 3·22389 *ghaṭikās*.
= 1160·60 *asus*.

(*iii*) Again at 3·22389 *ghaṭikās* after the instant of apparent conjunction—

The longitude of the sun = 0 sign 25° 32′ 54″.
The longitude of the moon = 0 sign 26° 23′ 31″.
The longitude of the node = 0 sign 29° 19′ 18″.
The moon's celestial latitude = −13′ 1″.
Long. of the sun from true equinox = 48° 49′ 1″.
Time elapsed since sunrise = 7549·18 *asus*.
Long. of the orient ecliptic point = 166° 10′ 31″.
Long. of the nonagesimal = 76° 10′ 31″.
Declination of the nonagesimal = 23° 15′ 54.

$Z' = +27' 53''$. $\odot - N = -27° 21' 30''$.

The corresponding parallax in long. = +1·8582 *ghaṭikās*

Parallax in long. for the middle of the eclipse
= ·67517 *gh*.

The third approximation to the half duration for the end of the eclipse = 3·31977 *gh*.
= 1195·12 *asus*.

(*iv*) Again at 3·31977 *ghaṭikās* after the instant of apparent conjunction—

The longitude of the sun = 0 sign 25° 32′ 59″.
The longitude of the moon = 0 sign 26° 24′ 51″.
The longitude of the node = 0 sign 29° 19′ 17″.
The moon's celestial latitude = −12′ 55″.
Long. of the sun from true equinox = 48° 49′ 6″.
Time elapsed since sunrise = 7583·70 *asus*.
Long. of the orient ecliptic point = 166° 42′ 11″.
Long. of the nonagesimal = 76° 42′ 11″.
Declination of the nonagesimal = 23° 19′ 4″.

$Z' = +31' 9''$; $\odot - N = -27° 53' 5''$.

The corresponding parallax in long. = +1·8707 *ghaṭikās*.

The fourth approximation to the half duration for the end of the eclipse = 3·3523 *ghaṭikās*.
= 1206·83 *asus*.

(*v*) Again at 3·3523 *ghaṭikās* after the instant of apparent conjunction—

The longitude of the sun = 0 sign 25° 33′ 1″.
The longitude of the moon = 0 sign 26° 27′ 18″.

The longitude of the node	=	0 sign 29° 19′ 17″.
The moon's celestial latitude	=	−12′ 43″.
Long. of the sun from true equinox	=	48° 49′ 8″.
Time elapsed since sunrise	=	7595·41 *asus*.
Long. of the orient ecliptic point	=	166° 52′ 56″.
Long. of the nonagesimal	=	76° 52′ 56″.
Declination of the nonagesimal	=	23° 20′ 4″.

$Z' = +32' 21''$; $\odot - N = 28° 3' 48''$.

The corresponding parallax in long. = 1·8317 *ghaṭikās*.

Parallax in long. for the middle of the eclipse
= ·67517 *gh*.

Mean half duration for the end of the eclipse
= 2·15674 *gh*.

The fifth approximation to the half duration for the end of the eclipse
= 3·35327 *gh*.

As the next step will lead to the same result, we are to take that the true half duration for the end of the eclipse = 3·35327 *ghaṭikās* = 80 *min.* nearly.

Now the Calcutta Mean Time for the instant of apparent conjunction
= 12 *hrs.* 43 *min* ;

∴ the instant of the end of the eclipse
= 14 *hrs.* 3 *min.* C. M. T.

Phenomena	C. M. T. calculated	C. M. T. observed or calculated from the *Conn. des Temps.*	Difference
Beginning of Eclipse	11 *hrs.* 32 *min.*	11 *hrs.* 21 *min.*	11 *min.* later
Geocentric Instant of ~~opposition~~ conjunction	12 *hrs.* 26 *min.*	12 *hrs.* 0 *min.*	26 *min.* later
End of Eclipse	14 *hrs.* 3 *min.*	13 *hrs.* 34 *min.*	29 *min.* later

This brings us practically to the end of the chapter on the solar eclipses of the *Khaṇḍakhādyaka* ; but Pṛthūdaka brings in two more problems, *viz.*, (*i*) to find the phase of the eclipse at any desired time and (*ii*) to find the time for any definite phase. His method is derived from the *Brāhmasphuṭa-siddhānta*, and is illustrated below.

Problem 1. To find the part of the sun eclipsed at Calcutta at 1 *ghaṭikā* after the beginning of the eclipse considered above.

Now the eclipse began at 2·98 *ghaṭikās* before the middle of the eclipse. Hence we have, at this time 1·98 *gh.* before the middle of the eclipse—

The long. of the sun	=	0 sign 25° 27′ 52″.
The long. of the moon	=	0 sign 25° 10′ 56″.
The long. of the node	=	0 sign 29° 19′ 24″.
The moon's celestial latitude	=	−19′ 31″.
Long. of the sun from true equinox	=	48° 43′ 59″.
Time elapsed since sunrise	=	5675·73 *asus*.
Long. of the orient ecliptic point	=	137° 57′ 23″.
Long. of the nonagesimal	=	47° 57′ 23″.
Declination of the nonagesimal	=	17° 34′ 52″.

$Z' = -5° 19' 39''$; $\odot - N = 46' 36''$.

The corresponding parallax in long. = −·05398 *ghaṭikās*.
The corresponding parallax in latitude = −4′ 32″.

Here, Moon − Sun = −16′ 56″, which is called the base of a right-angled traingle, while the apparent celestial latitude of the moon is called the perpendicular.

Now the difference of 16′ 56″ is decreased by parallax, as the moon and the sun are both east of the nonagesimal at the time. The alteration of its value according to the *Brahmasphuṭa Siddhānta* is expressed thus :—

$$\text{Altered value} = \frac{\text{Difference} \times \text{approximate half duration}}{\text{Half duration}}$$

$$= \frac{16' 56'' \times 1{\cdot}90881}{2{\cdot}98}$$

$$= 10' 51''.$$

Moon's apparent celestial latitude

$$= -19' 31'' - 4' 32''$$
$$= -24' 3''.$$

Hence according to Brahmagupta's rule, the apparent distance between the centres of the sun and the moon

$$= \sqrt{(10' 51'')^2 + (24' 3'')^2}$$
$$= 26' 33''.$$

Sum of the semidiameters

$$= 32' 52'';$$

∴ the part of the sun obscured

$= 6' \, 19''$.

Sun's diameter $= 31' \, 51''$;

∴ the phase of the eclipse in digits

$$= \frac{6' \, 19''}{31' \, 51''} \times 12$$

$= 2$ digits $23'$.

The above method of Brahmagupta does not appear to be satisfactory. A better method would have been like this:—

At the time under consideration, the parallax in longitude expressed in time $= \cdot 0598$ *ghaṭikās*.

Now 4 *ghaṭikās* corresponds to $\frac{1}{15}$ (difference of daily motions)

$= 3116''$,

∴ ·0598 *ghaṭikās* to $42''$;

∴ the apparent difference of longitudes

$= 16' \, 56'' - 42''$

$= 16' \, 12''$.

The apparent distance between the centres of the sun and the moon

$$= \sqrt{(16' \, 12'')^2 + (24' \, 3'')^2}$$

$= 29' \, 40''$;

∴ the part of the sun obscured $= 3' \, 12''$.

Problem 2. To find the time from the beginning of the eclipse when the part of the sun obscured will be $6' \, 19''$ (Pṛthūdaka's result).

This by the converse of Pṛthūdaka's process would lead to the answer, 1 *ghaṭikā*, after the beginning of the eclipse.

This finishes the fifth chapter of the Khaṇḍakhādyaka relating to Solar Eclipses.

CHAPTER VI

On the Rising and Setting of Planets.

Pṛthūdaka begins this chapter by taking up some stanzas from the eighth chapter, which are numbered below as 1, 2 and 3.

1. Multiply the 'sine' of the (*Śīghra*) anomaly by the 'sine' of the maximum *Śīghra* equation and divide by the 'sine' of the corresponding *Śīghra* equation, the result is the '*Śīghra* hypotenuse' when the (*Śīghra*) anomaly is half a circle, this *Śīghra* hypotenuse is equal to the radius diminished by the 'sine' of the maximum equation; when the anomaly is equal to the whole circle, the same is equal to the radius increased by the same 'sine' of the maximum equation.

The *Śīghra* hypotenuse spoken of here is EP (fig. on page 50), when SP or EM is taken to be R; and it is EM (fig. on page 53), when $ES=R$. In the figure on page 50 when $\angle PEM$ is a maximum, PM is its 'sine'; in the figure on page 53, when the $\angle MES$ is a maximum, SM is its 'sine.'

In the former figure, $EP = \dfrac{R \sin PMK \times PM}{R \sin PEM}$,

and in the latter figure, $EM = \dfrac{R \sin MSK \times SM}{R \sin MES}$.

This is equivalent to the 'sine rule' for a triangle in plane trigonometry. Brahmagupta is here seen to be the first person to give it in Indian mathematics.

In either case the *Śighra* hypotenuse means the distance of the planet from the earth according to some definite scale.

2. Four, two, eight, six and ten, multiplied by 10, are respectively the degrees of the longitudes of the nodes of Mars, Mercury, Jupiter, Venus and Saturn. Nine, twelve, six, twelve and twelve multiplied by 10, are respectively the minutes of their deviations from the ecliptic at the mean distance from the earth.

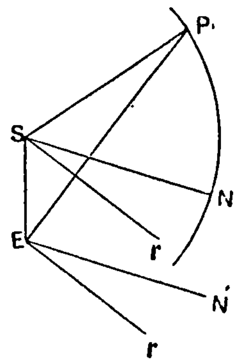
The import of the stanza may be thus exhibited :—

Planet	Longitude of node	Geocentric Orbital inclination
Mars	40°	90′
Mercury	20°	120′
Jupiter	80°	60′
Venus	60°	120′
Saturn	100°	120′

These figures are the same as in the *Aryabhaṭīya*, *Dasagītikā*, 8 and 9. Āmaraja's text would make the geocentric orbital inclination of Mercury to be 150′, which appears to be due to a misreading. According to Brahmagupta this is 152′, which converted to heliocentric inclination becomes 6° 20′, a result very near to the modern value.

3. From the apparent (heliocentric) longitude of a planet subtract that of the node, and in the case of Mercury and Venus subtract the longitude of the node from the *Śīghrocca* (*i.e.*, the mean heliocentric longitude); the 'sine' of the remainder multiplied by the deviation and divided by the *Śīghra* hypotenuse of the last operation is the celestial latitude in minutes.

In this stanza the word *samalipta* means 'either of the two planets which have the same geocentric longitude. This is wrong. In the case of a superior planet it is necessary to use the heliocentric longitude, *i.e.*, the geocentric longitude *minus* the annual parallax. In the figure given here, let E, S, P, be the positions of the earth sun and a superior planet, SN the line of nodes; here the angle on which the celestial latitude depends is the angle PSN. Let Sr and Er be the direction of the first point of Aries. Through E, draw EN' parallel to SN.



From stanza 2, we get the angle rEN', which is equal to rSN.

$$\text{Now, } \angle PSN = \angle rSP - \angle rSN$$
$$= \angle rEP - \angle EPS - \angle rEN'$$
$$= \text{Geocentric Long.} - (\text{Long. of node} + \textit{Śighra Phala})^*$$

The heliocentric celestial latitude of P

$$= \frac{\text{Orbital inclination} \times SP \sin PSN}{SP}, \text{ according to the Siddhantas.}$$

Hence the geocentric celestial latitude

$$= \frac{\text{Orbital inclination} \times SP \sin PSN}{SP} \times \frac{SP}{EP},$$

$$= \frac{\text{Orbital inclination} \times SP \sin PSN}{EP}.$$

Here $EP = R$ and EP is H the *Śighrakarṇa* as explained before.

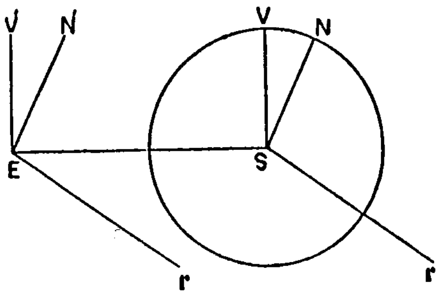
Thus the geocentric celestial latitude

$$= \frac{\text{Orb. inclination} \times R \sin \{\text{Geo. long.} - (\text{long. of node} + \textit{Śighraphala})\}}{H}$$

Hence it is clear that from the apparent geocentric longitude of a planet it is necessary to subtract the *Śīghra* equation or the annual parallax of the fourth step in finding the apparent longitude in the case of a superior planet.

* *Surya Siddhānta*, II, 56 and 5.

In the case of an inferior planet the treatment is different. Let E, V, S be the positions of the earth, Venus and the centre of the epicycle (here the real orbit) of Venus. Here also let SN be the line of nodes; Er, and Sr are the directions of the first point of Aries; EV' and EN' are parallel to SV and SN. The celestial latitude depends on $\angle VSN$.



Now $\angle VSN = \angle VSr - NSr$

$= \angle V'Er - \angle N'Er$

$=$ *Śīghra* of Venus $-$ Node of Venus.

The heliocentric celestial latitude of V

$$= \frac{\text{Heliocentric orbital inclination} \times R \sin VSN}{R}$$

The geocentric celestial latitude is therefore

$$= \frac{\text{Heliocentric orbital inclination} \times R \sin VSN \times p}{R \times EV}$$

Where $p = SV$; $EV = H$, the *Śīghra* hypotenuse.

The stanza gives the rule as equivalent to this—

Geocentric celestial latitude

$$= \frac{\text{Geocentric orbital inclination} \times R \sin VSN}{EV}.$$

Now the Geocentric orbital inclination of Venus is given to be $= 120'$;

$$\therefore 120' = \frac{\text{Heliocentric orbital inclination} \times P}{R};$$

but $\frac{P}{R} = \frac{260}{360}$.

Hence the heliocentric orbital inclination of Venus

$= \frac{120' \times 18}{13} = 2° 46' 9''$, the modern value being about 3° 23' 37″.

The rule of the stanza in the case of an inferior planet is quite correct. In the case of a superior planet, however, the geocentric apparent longitude has to be converted into the heliocentric longitude by applying the *Sighra* equation or annual parallax in the inverse order. We now illustrate the rules by an example.

Illustration.—Let the time be Śaka year 1851, 11 synodic months and 19 *tithis* or A.D. 1930, the 18th March; to examine if Venus was heliacally visible.

The *ahargaṇa* = 462024 at midnight on Tuesday at Ujjayinī

The mean Venus or the mean sun = 11 signs 2° 37′ 43″.

The *Śīghra* of Venus = 0 sign 11° 40′ 31″

Lalla's correction at −153′ per year to the *Śīghra* of Venus from 421 of Saka year = −15°;

∴ the longitude of the *Śīghra* of Venus

= 11 signs 26° 40′ 31″.

The long. of Venus' and sun's apogee = 2 signs 20°

The sun's equation = 134′ sin 72° 37′ 43″

= + 2° 7′ 53″.

The sun's apparent longitude = 11 signs 4° 45′ 36″.

Śīghra anomaly = 24° 2′ 48″,

Venus' *Śīghra* equation = 10° 3′ 25″.

This halved = 5° 1′ 43″.

The new mean Venus for the 2nd step = 11 signs 7° 39′ 26″

Venus equation of apsis = 134′ sin 77° 39′ 26″

= +2° 10′ 54″.

This halved = +1° 5′ 27″.

The mean Venus for the equation of apsis of the 3rd step

= 11 signs 8° 44′ 53″.

Venus' equation of apsis of the 3rd step

= 134′ sin 78° 44′ 53″

= + 2° 11′ 26″.

Mean Venus of the 1st step = 11 signs 2° 37′ 43″.

Venus as corrected by the equation of apis, which is the same as the centre of the epicycle of Venus, = 11 signs 4° 49′ 9″.

Venus Sighra equation = 9° 12′ 40″

Geocentric Venus = 11 signs 14° 1′ 49″.

Sun's apparent longitude = 11 signs 4° 45′ 36″.

Difference = 9° 16′ 13″

which represents Venus' elongation on that day, and according to the *Āryabhaṭīya*, *Golā*, 4, Venus ought to be heliacally visible.

But Brahmagupta would take this 9° as *kālāṅśa* or 36 min. of time as the interval between the setting of the sun and of Venus as the time criterion for the visibility of Venus by the naked eye.

We are thus to find the times of setting of sun and Venus on the same day, say at Calcutta, and have to use the longitudes of these two bodies at sun set on the same day, *i.e.*, at 7 hours 18 min. of Ujjayinī time. These by the method described above are the following:—

The sun's longitude	= 11 signs 4° 28′ 55″.
Venus' ,,	= 11 signs 13° 30′ 30″.
The final *Śīghra* anomaly	= 21° 35′ 46″.
,, Equation	= 9° 4′ 1″.
'Sine' of max. *Śīghra* equation	= 108 p. 20′.
Radius	= 150 p.
The *Śīghra* hypotenuse	$= \frac{150 \sin 21° 35' 4\,'' \times 108}{150 \sin 9° 4' 1''}$
	= 253·022.
The *Śīghra* of Venus	= 11 signs 26° 8′ 15″.
Long. of the node of Venus	= 2 signs 0° 0′ 0″.
Difference	= 9 signs 26° 8′ 15″;

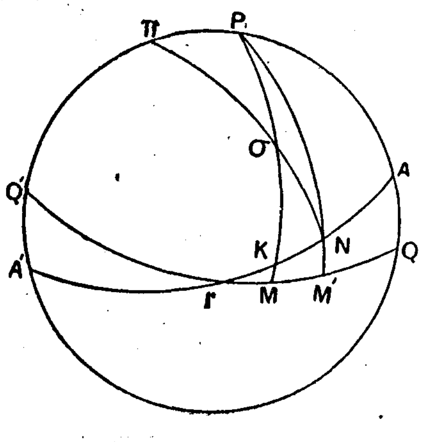
∴ geocentric celestial latitude of Venus

$$= -\frac{120' \times \sin 63° 51' 45'' \times 150}{253{\cdot}022}$$

$$= -\ 1° 8' 39''.$$

This illustrates the rule for finding the celestial lalitudes of planets. The next stanza describes the conversion of celestial longitudes into polar longitudes.

In the adjoining figure let σ be the position of a heavenly body of which rN and σN are the celestial longitude and the celestial latitude of σ. Let Q′rQ be the celestial equator, A′rA the ecliptic; let P and π be their poles; $\pi\sigma N$ a secondary to the ecliptic, $r\sigma KM$ and $BRNM$ the secondaries to the equator, σR perpendicular to PNM. The aim is to find KN which subtracted from rN or added to it gives rK the polar longitude', and σK is called the 'polar



latitude.' The older astronomers attempted to find σR* and they equated it to KN. Thus we have:—

4. Multiply the celestial latitude by the 'sine' of the longitude increased by three signs (*i.e.*, 90°) and divide by 371; subtract the resulting minutes from the longitude according as the sun's course in which the body is and the celestial latitude, are of the same denomination and add the result to the longitude if they are of different denominations.

In the triangle πPN,

$$\text{Sin } \pi NP = \frac{\sin \pi P \times \sin P}{\sin 90°}.$$

Here $\angle P = M'Q$, has been practically taken equal to NA.

$$\text{Hence } R \sin \pi NP = \frac{R \sin 24° \times R \cos rN}{R}$$

$$= \frac{\sigma R \sin 24° \times R \sin (90° + rN)}{R};$$

$$\therefore \sigma R = \frac{\sigma N \times R \sin \pi NP}{R}$$

$$= \frac{\sigma N \times R \sin 24° \times R \sin (90° + rN)}{R \times R}$$

$$= \frac{\sigma N \times R \sin (90° + rN)}{\frac{R^2}{R \sin 24°}}.$$

In the *Khaṇḍakhādyaka*, $R = 150$; and Brahmagupta here takes,

$$\frac{R^2}{R \sin 24°} = 371;$$

thus he took $R \sin 24° = 60\ p\ 40'$.

Hence $\sigma R = \frac{\sigma N \times R \sin (90° + rN)}{371}$, which is taken to be KN.

* R is the foot of the perp from σ on the arc PN.

This proves Brahmagupta's rule. It is a distinct improvement upon Āryabhaṭa.* As to how $R \sin \cdot (90° + rN)$ occurs in this equation, has been indicated by Bhaskara II, *Golādhyāya*, VIII, Comm. on 30-74, illustration 1, which has been detailed in full in the translator's paper "Greek and Ḥindu Methods in Spherical Astronomy," Problem V.

Illustration.—In the example taken, as found already

The geocentric longitude of Venus $= 11$ signs $13° 36' 30''$.

The geocentric celes. lat. of Venus $= -1° 8' 39''$.

The total shifting of the equinoxes from 421 of the *Śaka* era

$= 23° 17' 8''$;

$\therefore$ rN in this case $= 0$ sign $6° 53' 38''$,

and σN $= -1° 8' 39''$.

The correction to the celestial long. of Venus for polar longitude

$$= \frac{68'{\cdot}65 \times 150 \sin 96° 53' 38'}{371}$$

$$= +25' 38''.$$

The polar longitude of Venus $= 7° 19' 16''$.

The polar latitude of Venus $= -1° 8' 39''$.

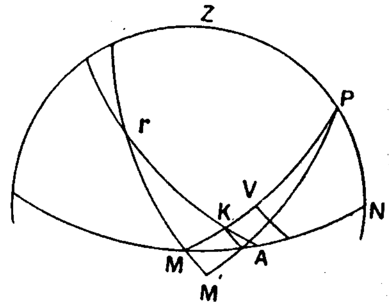
The point of the ecliptic which will set simultaneously with Venus, at Calcutta, has now to be found when the point of the ecliptic at $7° 19' 16''$ will be on the western horizon, Venus having a south-polar latitude will have already set. Hence a correction will have to be applied to the polar longitude to find this required point. We now have the following rule for this purpose.

5. Multiply the north celestial latitude by the equinoctial shadow and divide by 12 ; apply the quotient taken as minutes *negatively* and *positively* to the orient and occident ecliptic points. When the celestial latitude is south, apply the resulting minutes to the same points positively and negatively.

* *Gola.*, 36, where in place of $R \sin (90° + rN)$, R vers rN is used. Strangely enough Āryabhaṭa's wrong rule is preferred to this rule of Brahmagupta, by Lalla, Pṛthūdaka and even by Āmarāja (1102 of *Śaka* year or 1130 A.D.).

If λ be the polar latitude and $12 \tan \phi$ the equinoctial shadow for the station of latitude ϕ, the correction is

$$= \frac{\lambda \times 12 \tan \phi}{12} \; min.$$



Let $NPZQ$ be the observer's meridian, NM the horizon, M the east point, QM the equator, rKA a part of the ecliptic. Let V be the position of a heavenly body, of which rK is the polar longitude, and VK the polar latitude. When K is on the six o'clock circle, the part of the diurnal circle of K intercepted between this circle and the horizon is roughly

$$= \frac{MK \times 12 \tan \phi}{12}.$$

Similarly the part of the diurnal circle of V, between the six o'clock circle and the horizon,

$$= \frac{MV \times 12 \tan \phi}{12} \text{ roughly.}$$

Their difference $= \frac{KV \times 12 \tan \phi}{12}$, which is here approximately taken to be equal to the negative correction to rK. If KV were south the correction would be positively applied to rK.

On the western horizon, when KV is north the correction is applied positively and when south, it is applied negatively.

Illustration.—In the example selected KV is south, at Calcutta the latitude $\phi = 22° 35'$, and $KV = -1° 8' 39''$.

The correction $= -68' 39'' \tan \phi$

$= -26' 34''$.

Hence the point of the ecliptic which will set simultaneously with Venus $= rK - 26' 34''$

$= 7° 19' 16'' - 26' 34''$

$= 6° 52' 42''$.

The sun's longitude $= 11$ signs $4° 28' 55''$.

The sun's long. from true equinox $= 357° 46' 3''$.

Long. of the orient ecliptic point at sunset

$= 177° 46' 3''$.

Long. of the orient ecliptic point when Venus sets

= 186° 52′ 42″.

Difference = 9° 6′ 39″

of Libra and Virgo, each of which rises in 1966 *asus* at Calcutta.

Hence this difference of 9° 6′ 39″ rises at Calcutta in 597 *asus* = 9° 57′, which represents what is called *kālāṁśa* or degrees indicative of time. The next stanza speaks of these *kālāṁśas*. The two operations described above are called the *Dṛkkarmas*.

6. Venus, Jupiter, Mercury, Saturn and Mars, corrected by ***Dṛkkarma*** become visible (heliacally) when separated from the sun by the *Kālāṁśa* of 9 increasing by a common difference of 2, and they are invisible if separated by less; the moon is visible if separated by 12 of ***Kālāṁśa*** from the sun.

The rule may be exhibited as follows:

Planet	Degrees of *Kālāṁśa* from the sun necessary for visibility
Venus	9°
Jupiter	11°
Mercury	13°
Saturn	15°
Mars	17°
Moon	12°

This stanza is comparable with that of the *Āryabhaṭīya*, *Gola*, 4, the only difference being the use of the term *Kālāṁśa* in place of difference of longitudes.

Illustration.—In our example Venus was separated from the sun by 9° 57′ of *Kālāṁśa*, at sunset at Calcutta on the 18th March, 1930. Hence Venus became first visible by the naked eye at evening at Calcutta on that date according to the *Khaṇḍakhādyaka*, the difference between the settings of the sun and Venus being 39 *mins.* 48 *secs*.

7. When the planet's rising ecliptic point has a less longitude than the orient ecliptic point, the planet has already risen, when greater the planet is yet to rise. When the planet's setting ecliptic point increased by six signs is less than the orient point, it has already set when greater the planet is yet to set. The intervening *ghaṭikās* of time are obtained by making the less equal to the greater by means of the local time intervals for the risings of the signs of the zodiac.

This stanza requires no explanation and has been already illustrated. We work out one more problem relating to heliacal rising.

Problem. To examine if Jupiter was heliacally visible in the east on the 5th July, 1930, before sunrise.

The time is *Śaka* year 1852, 3 Synodic months 9 *tithis* elapsed, the sunrise being at 5 *hrs.* 21 *mins.* Calcutta time, which is 4 *hrs.* 30 *mins.* of Ujjayinī time.

The mean Sun and *Śīghra* of Jupiter	= 2 signs 19° 15′ 30″.
The mean Jupiter	= 2 signs 5° 54′ 59″.
Lalla's correction to mean Jupiter	= −4° 29′ 2″.
The corrected mean Jupiter	= 2 signs 1° 25′ 57″.
The Sun's equation	= +134′ sin 44′ 30″.,
	= 1′ 44″.
The Sun's longitude	= 2 signs 19° 17′ 14″.
Jupiter's longitude as corrected by the equation of apsis	= 2 signs 6° 31′ 16″.
Jupiter's geocentric longitude	= 2 signs 8° 47′ 58″.
Long. of Jupiter's ascending node	= 2 signs 20°.
Jupiter's *Śīghra* hypotenuse	= 179 *p*.
Jupiter's celestial latitude	= −11′ 42″.
Total shifting of the equinoxes	= 23° 17′ 23″.
Sun's apparent longitude	= 102° 34′ 37″.
Jupiter's geocentric longitude	= 92° 5′ 21″.
,, ,, celestial latitude	= −11′ 42″.

The first *Dṛkkarma* correction to

$$\text{Jupiter's longitude} = \frac{+11'42'' \times 150 \sin 182^\circ 5' 21''}{371},$$

$$= -10''.$$

The second *Dṛkkarma* correction to Jupiter's longitudes

$$= \frac{+11' 42'' \times 12 \tan 22° 35'}{12}$$

$$= +4' 30''.$$

Jupiter's long. as corrected by the two *Dṛkkarma* operations

$$= 92° 9' 41''.$$

Sun's longitude $= 102° 34' 37''.$

Difference $= 10° 24' 56''$

which is in the sign of Cancer which rises in 2037 *asus*.

The difference in *Kālāṁśa's* $= \frac{2037' \times 10° 24' 56''}{30°},$

$$= 707'$$

$$= 11° 47'.$$

Now the *Kālāṁśa* for the heliacal rising of Jupiter is 11° according to the *Khaṇḍakhādyaka*. Thus Jupiter was heliacally visible on the morning of the 5th July, 1930, at Calcutta.

Similarly may be worked out other cases of heliacal risings of the moon and of other 'star' planets.

This finishes Chapter VI of the Khaṇḍakhādyaka, which relates to the rising and setting of heavenly bodies.

CHAPTER VII

On the Position of the Moon's Cusps.

1. 703′, 535′ and 202′ are the minutes of the tabular differences of the declinations of the last points of Aries, Taurus and Gemini. By means of the moon's apparent declination from which as many as possible of these parts have been subtracted, the moon's corresponding ascensional difference is obtained as usual by adding up the corresponding integral and fractional parts of the tabular differences passed over.

In Chapter III, stanza 1, are given the tabular ascensional differences for one, two and three signs in *bināḍīs* as—

$\frac{159}{16} \times 12 \tan \phi$, $\frac{65}{8} \times 12 \tan \phi$, $\frac{10}{3} \times 12 \tan \phi$, respectively for the last points of Aries, Taurus and Gemini, corresponding to the declinations—

703′, 1238′ and 1440′.

In the case of the moon proportional parts cannot be taken by using the longitude; hence Brahmagupta here lays down very correctly the rule that the moon's ascensional difference should be found from the apparent declinations. The ascensional difference found from this rule would not, however, be very accurate. Again if the obliquity of the ecliptic be taken = 24° and the inclination of the moon's orbit at 4° 30′, the maximum declination of the moon may come to about 28° 30′, for which the present rule is quite inadequate.

To illustrate the methods of this chapter we propose to examine the position of the moon's cusps on the 28th June, 1930, or 1852 of the *Saka* year, 3 synodic months and 2 *tithis*, at 17 *hrs.* 56 *mins.* of Ujjayinī time, which is the time of sunset at Calcutta on that day.

Here the *ahargaṇa* $= 462125 + \frac{179}{240}$,

$= 462125 + \frac{3}{4} - \frac{1}{4 \times 60}$

The mean sun = 2 signs 12° 54′ 33″.
The mean moon = 3 signs 10° 49′ 59″.
Moon's apogee with Lalla's correction = 9 signs 23° 10′ 46″.
Sun's apogee = 2 signs 20°.
Moon's ascending node with Lalla's correction
= 0 sign 6° 24′ 19″.
The sun's equation = +16′ 32″.
The moon's equation = −12′ 8″.
The apparent sun = 2 signs 13° 11′ 5″.
The apparent moon = 3 signs 10° 37′ 51″.
The moon's celestial latitude = +4° 29′ 14″.
Total shifting of the equinoxes = 23° 17′ 23″.
The sun's apparent longitude = 96° 28′ 28″.
The moon's apparent longitude = 123° 55′ 14″.
The moon's apparent declination = 28° 12′ 45″.
Latitude of Calcutta = 22° 35′.
The moon's ascensional difference as worked out by the formula
= 10° 16′ 28″
= 102 *bin.* $44\frac{4}{5}$ *bip.*
The same as worked by the parts = 102 *bin.* 55 *bip.*
The moon's semidiurnal arc = 100° 35′ 14″.
The sun's semidiurnal arc * = 100° 35′ 14″.
The moon's celestial latitude = +4° 29′ 14″;
∴ the moon's polar longitude = 124° 55′ 59″.
The sun's apparent longitude = 96° 28′ 28″;
∴ the difference of the right ascensions of the sun and the moon
= 30° 12′.
The moon's hour angle = 70° 23′ 14″ W.,

neglecting the moon's parallax and also the refraction which latter was not detected by the makers of Indian Astronomy.

The moon's declination = 23° 12′ 45″;
∴ the moon's zenith distance = 64° 8′ 17″.
The sun's declination = 23° 50′ 14″;
∴ the sun's azimuth from the West = 25° 57′ 26″N.

* Or the sun's hour-angle at sunset.

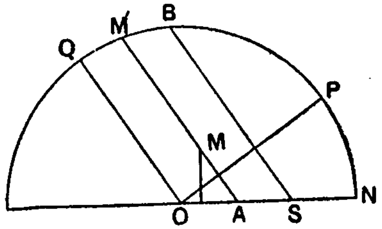
The moon's *Saṁkutala* = 27*p* 12′ 50″.
The moon's *Agrā* = 64*p* 49′ 2″.
The moon's azimuth from the West = 15° 49′ 25″N.
The difference of the azimuth's of the sun and the moon
= 10° 8′ 1″.
The arc between sun and the moon = 27° 47″;
∴ the angle between the line of the cusps and the horizontal
= 22° 10′ 32″,

which is the angle by which the northern cusp is elevated.

We have thus shown above the modern way of solving the problem and proceed presently to illustrate the method of the *Khaṇḍakhādyaka*, which is also the method of the Indian *Siddhāntas*. It must be stated at the outset that the makers of the Indian *Siddhāntas* could not really solve the problem, and were satisfied with only an approximation. Further, the rule given above for finding the moon's ascensional differences is really unnecessary and misleading. The calculation of the orient ecliptic points for the rising and setting of the moon and thus to arrive at the sidereal measure of the length of the moon's day is equally useless and cannot help us in finding the moon's hour-angle. Pṛthūdaka's Sanskrit commentary uses this method for details of which the reader is referred to his commentary.

2-3. Take the 'sine' of the difference or the sum of the declinations of the sun and the moon according as they are of the same or opposite denominations, multiply it by the hypotenuse of the gnomonic shadow triangle and divide by the 'sine' of the co-latitude; add this to the equinoctial shadow if the result be of the same denomination with it, or lessen it by the equinoctial shadow if they are of opposite denominations; the result is the perpendicular and is south from the place where the moon is; the base is twelve digits, and the square root of the sum of their squares is the hypotenuse.

Let the positions of the sun and the moon be projected on the meridian plane. In our illustrative example the sun is on the horizon. Let O be the position of the observer; OP the line joining the observer and the celestial pole; OQ, $M'MA$ and BS are traces of the celestial equator and the diurnal circles of the moon and the sun respectively. From M, the projection of the position of the moon on the meridian plane let MK be drawn perpendicular to the north-south line. The stanzas aim at finding a triangle similar to MKS in the figure.



Here $KS = KA + AS,$

$= KA + OS - OA,$

$=$ Moon's *Saṁkutala* + Sun's *Agrā* − Moon's *Agrā*.

Now *Saṁkutala* $= \dfrac{R \cos Z \times R \sin \phi}{R \cos \phi}$, where Z is the moon's zenith distance, and ϕ the latitude of the station.

The sun's *Agrā* $= \dfrac{R \sin \delta' \times R}{R \cos \phi}$, where δ' is the sun's declination.

The moon's *Agrā* $= \dfrac{R \sin \delta \times R}{R \cos \phi}$, where δ is the moon's declination.

$$\text{Hence } KS = \frac{R \cos Z \times R \sin \phi}{R \cos \phi} + \frac{R \sin \delta' \times R}{R \cos \phi} - \frac{R \sin \delta \times R}{R \cos \phi}.$$

Now in a triangle similar to MKS, the side corresponding to MK is to made = 12 digits and MK is $R \cos Z$. Hence the side corresponding to KS should be $\dfrac{KS}{MK} \times 12 = \dfrac{12 \times KS}{R \cos Z}$, which is called here the perpendicular or *Bhuja*;

$$\therefore \quad Bhuja = 12 \tan \phi + \frac{12 \times R}{R \cos Z} \times \frac{R \sin \delta' - R \sin \delta}{R \cos \phi}.$$

The expression $\dfrac{12\ R}{R \cos Z}$, where Z is the moon's zenith distance, is called the hypotenuse of the moon's gnomonic shadow triangle; $12 \tan \phi$ is the sun's equinoctial noon shadow. In the rule in place of $R \sin \delta' - R \sin \delta$, Brahmagupta gives here $R \sin (\delta' - \delta)$; but in his *Brāhmasphuṭa-siddhānta*, VII, 6, his rule agrees with the expression obtained above.

Now the *Koṭi* (*i.e.*, corresponding to MK) = 12;

$\therefore$ the hypotenuse $= \sqrt{(Koṭi)^2 + (Bhuja)^2}$,

$= \sqrt{12^2 + (Bhuja)^2}$,

which is the expression for the hypotenuse in the rule.

As to the direction of the *Bhuja*, it is south from S in the figure, when the moon is south from the sun, and it is north from S in the figure when the moon is north from the sun.

Illustration.—In the problem proposed,

$$\text{the } Bhuja = 12 \tan 22° \; 35' + \frac{12 \; (\sin 23° \; 50' \; 14'' - \sin 23° \; 12' \; 45'')}{\cos 64° \; 8' \; 17'' \times \cos 22° \; 35'},$$

$= 7p \; 34' \; 54''$.

Koṭi $= 12p$;

$\therefore$ the hypotenuse $= 13p \; 33' \; 19''$.

The angle $KMS = 32° \; 17'$ nearly, this will be as we shall see later on, the elevation of the northern cusp according to the rule.

4. The difference in degrees between the longitudes of the moon and of the sun divided by 15, gives the measure of the illuminated portion along the hypotenuse as calculated before. The obscured part is found as in the case of the sun in the disc of the moon which is taken as of 12 digits.

The moon's diameter = 12 digits. When the angular distance from the sun = 180°, the whole disc is illuminated; for any angular distance the maximum breadth of the illuminated portion is found by the expression

$$\frac{12 \times \text{angular distance from the sun in degrees}}{180°} \text{ digits}$$

$$= \frac{\text{angular distance in degrees}}{15} \text{ digits.}$$

A better approximation would have been

$$= \frac{6 \times R \text{ vers (angular distance from the sun)}}{R} \text{ digits,}$$

which is given by Lalla in *Śiṣyadhibṛddhida*, IX, 12; Brahmagupta also accepts this rule in his *Brāhmasphuṭa-siddhānta*, VII, 11, as an alternative process.

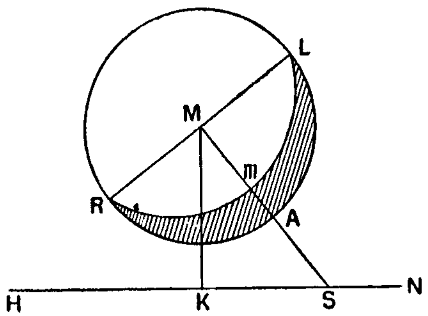
The maximum breadth of the illuminated portion of the disc being determined from this rule, this is to be given in the disc of the moon along the hypotenuse, the point thus arrived at being called *Sitāsita* point (*i.e.*, a point showing the boundary separating the lighted and dark portions of the moon) by Pṛthūdaka. The method of finding the dark portion of the moon appears to be the following:—From the vertex of the triangle where the hypotenuse and the *Koṭi* meet, a line is drawn perpendicular to the hypotenuse. This cuts the circumference of the disc into two points. The circle described through these two points and the *sitāsita* point is the line of demarcation of the lighted and dark portions of the moon's disc.* Pṛthūdaka speaks of a different process, which is this—describe another circle of the same radius that of the disc of the moon through the *sitāsita* point, the centre of the circle lying on the hypotenuse produced (*i.e.*, in the direction in which the *sitāsita* point is marked from the boundary of the disc).

Illustration.—We have found before that the perpendicular or *Bhuja* = 7p 34′ 57″.

Koṭi = 12p.

Hypotenuse = 13p 33′ 19″.

Let *NH* be the north-south line; take any point *S* in it: from *S* measure *SK* in the direction in which the moon is from the sun and equal to 7p 34′ 57″ or the *Bhuja*; with *K* and *S* as centres draw two circles with radii equal respectively to *Koṭi* (here 12p) and the hypotenuse (here 13p 33′ 19″), intersecting each other at *M*.



With *M* as the centre and radius = 6p, describe a circle representing the moon's disc, cutting *MS* at *A*.

* Cf. *Brāhmasphuṭa-siddhānta*, VII, 14; *Sūrya-siddhānta*, X, 7-11; *Śiṣyadhibṛddhida*, IX, 14-16; *Pañca-siddhāntikā*, V, 7.

Now, Moon − Sun = 27° 30′;

∴ the maximum breadth of the illuminated portion

$$= \frac{27° 30'}{15°} = 1p\ 50'.$$

Measure off *Am* along *AM* = 1p 50′. Through *M* draw *LMR* perpendicular to *SM*, cutting the circle representing the disc at *L* and *R*. Now draw a circle through *L*, *m* and *R*; now the shaded portion represents the moon's figure. As shown before the elevation of *L* = 32° 17′, while we found this to be 22° 10′ 32″ nearly.

The *Siddhānta*-makers apparently failed to recognise that the line of cusps is at right angles to the line joining the observer and the moon as also to the line joining the moon and the sun. The further details given in Pṛthūdaka's commentary are not interesting.

This brings us to the end of Chapter VII, which relates to finding the position of the moon's cusps.

CHAPTER VIII

On Conjunction of Planets.

1. Four, two, eight, six and ten, multiplied by ten, are the numbers of degrees in the longitudes of the nodes planets beginning with Mars. Nine, twelve, six, twelve and twelve, multiplied by ten, are the minutes of the deviations from the ecliptic of the same at the mean distance from the earth.

This stanza has been considered in Chapter VI, as stanza 2.

2. The 'sine' of the *Śīghra* anomaly multiplied by the 'sine' of the maximum *Śīghra* equation and divided by the 'sine' of the *Śīghra* equation is the *Śīghra* hypotenuse. When the *Śīghra* anomaly is half a circle, the hypotenuse equals the radius diminished by the 'sine' of the maximum equation; when the *Śīghra* anomaly equals the whole circle the same increased by the same 'sine' is the hypotenuse.

This also has been considered in Chapter VI, as stanza 1.

3. Divide the interval between two planets by the difference of their daily motions when they are moving in the same direction or by the sum of their daily motions when they are moving in opposite directions; the quotient taken as days represents the time, by which the conjunction is to come when the slower is ahead of the quicker, and by which it is over if the quicker is ahead.

The concluding portion is incomplete in as much as it does not state how to find if the conjunction is to come or over when two planets are moving in opposite direction.

4. The interval between two planets multiplied by one of the planets' own daily motion and divided as before (as directed in the preceding stanza) is applied negatively when the conjunction is over and positively when it is to come; this would make the planets of equal longitude. For the planet having a retrograde motion, the negative and positive applications are to be made in the reverse order.

This stanza also requires no explanation.

5. From the planets which have been thus made of equal longitude, subtract the respective ascending nodes; and in the cases of Mercury and Venus subtract the nodes from their *Śīghroccas*: take the 'sine' of the resulting arc; multiply it by the minutes of the mean deviation from the ecliptic and divide by the *śīghra* hypotenuse of the last operation; the final result is the celestial latitude of the planet.

This stanza has been already considered as stanza 3 of Chapter VI.

6 (1st half). Of two planets which have the same longitude, the difference of their celestial longitudes when they are of the same name is the distance between them; when the celestial latitudes are of different names their sum is the distance between them.

In this case also no explanation is necessary. The distance between two planets used to be measured in *hastas*; one *hasta* was taken equal to 1° or 60′ of arc; hence 1 *aṅguli* = 2′ 30″.

6 (2nd half). When two planets' centres coincide, the rest of the calculation is the same as in the case of a solar eclipse. The celestial latitude of the lower planet is to be corrected by the parallax in latitude as in the case of the moon.

Here no detailed explanation is necessary, excepting that the necessary changes in the constants for the parallaxes and semi-diameters of the planets will have to be made. To illustrate this second half of the stanza by a concrete example is also not easily available or possible. We may try to verify the conjunction of Venus and Jupiter on the 17th May, 1930, at 18 *hrs*. G.M.T., *Con. des Temps*. 1930, page 560.

Illustration.—Time, *Śaka* 1852, 1 synodic month, 19 *tithis* elapsed; the day of the week being Saturday.

The *ahargaṇa* up to the end of Saturday at Ujjayinī midnight $=462084$.

For the conjunction we have to subtract $\frac{3}{80}$th of a day from the *ahargaṇa*, *i.e.*, the *ahargaṇa* is to be taken $=462084-\frac{3}{80}$. At this *ahargaṇa*—

The mean sun, the mean Venus or Jupiter's *Śīghra*	$=$ 1 sign 1° 43′ 40″.
The mean Jupiter	$=$ 2 signs 1° 50′ 20″.
Lalla's correction to Jupiter	$=$ $-4°$ 29′ 2″;
∴ the mean Jupiter	$=$ 1 sign 25° 21′ 18″.
The *Śīghra* of Venus	$=$ 3 signs 17° 19′ 25″.
Lalla's correction thereto	$=$ $-14°$ 30′ 46″.
The *Śīghra* of Venus	$=$ 3 signs 20° 48′ 39″.
The long. of Venus' apogee	$=$ 2 signs 20°.
The long. of Jupiter's apogee	$=$ 5 signs 10°.
The long. of Venus' ascending node	$=$ 2 signs.
The long. of the Jupiter's asc. node	$=$ 2 signs 20°.
Venus' geocentric longitude	$=$ 1 sign 27° 37′ 19″.
Venus' geocentric latitude	$=$ 0° 43′ 22″N.
Jupiter's geocentric longitude	$=$ 1 sign 25° 39′ 10″.

Hence according to the *Khaṇḍakhādyaka* constants with Lalla's correction the conjunction should have happened two days before. This is most probably due to the fact that Lalla's correction as applied here in the case of Jupiter is an over-correction.

In the *Brāhmasphuṭa-siddhānta* this topic is considered under three aspects:—

(*i*) The conjunction by the equality of geocentric celestial longitudes.

(*ii*) The conjunction by the equality of polar longitudes or of right ascensions.

(*iii*) The conjunction by the simultaneous position on the same secondary to the prime vertical.

In the *Khaṇḍakhādyaka* as we have just now seen it is the first aspect that is considered. The second half of the last stanza speaks of what is known as *Bhedayuti*. This would include transits of Venus and Mercury on the sun's disc.

The *Khaṇḍakhādyaka* as originally composed by Brahmagupta consisted of two parts:—

(*i*) In the *Khaṇḍakhādyaka* proper the astronomical constants used are all according to the teachings of Āryabhaṭa I in his system of astronomy referred to by all writers and commentators as the *Ārdharātrika* system. This part consists of eight chapters, this being the last.

(*ii*) The *Uttara Khaṇḍakhādyaka*, which contained Brahmagupta's corrections to the first part and other improvements as also supplementary chapters.

Pṛthūdaka's text gives some of the stanzas of this *Uttara* portion which will be considered in the next chapter. An attempt will also be made to reconstruct some of the missing chapters from Bhaṭṭotpala's text from which Alberuni makes profuse quotations. The Berlin manuscript which we are using for this edition breaks up abruptly. Bhaṭṭotpala's text is also unreliable.

This brings us to the end of the Khaṇḍakhādyaka proper, the last chapter relating to the Conjunction of Planets.

CHAPTER IX

Corrections and New Methods.

* चक्रतार्य्यभटः शीघ्रगमिन्दूच्चं पातमल्पगं स्वगतेः ।
तिथ्यन्तग्रहणानां घुणाक्षरं तस्य संवादः ॥१॥

1. Āryabhaṭa made the apogee of the moon as moving more quickly and the node as moving more slowly than their actual motions; if his constants give correct results in relation to the end of *tithis* (*i.e.*, conjunction, etc.) or eclipses they must be considered as accidental as are the letters cut into wood by weevils.

The following is a comparative statement of the sidereal periods of the moon's apogee and the ascending node:—

Sidereal Periods:—

	According to *Aryabhaṭiya*	According to *Khaṇḍakhādyaka*	Modern Values
Moon's apogee ...	3231·987079 da.	3231·987844 da.	3232·3754 da. (Lockyer)
Moon's node ...	6794·749511 da.	6794·750834 da.	6793·39108 da. (Lockyer)

Hence Brahmagupta is not wrong in his contention; we shall presently consider how far he himself is correct in finding the correct sidereal periods.

† मध्यगतिज्ञं दृष्ट्वा श्रीषेणार्य्यभट-विष्णुचन्द्रज्ञाः ।
सदसि न भवन्त्यभिमुखाः सिंहं दृष्ट्वा यथा हरिणाः ॥२॥

* Taken from the *Brāhma-sphuṭa-siddhānta*, I, 61.

† " " " " " " , I, 52.

2. On seeing me, who possess the most accurate knowledge of mean motions, men who have learnt from the works of Śrīṣeṇa, Viṣṇucandra and Āryabhaṭa, cannot face me in any meeting just like deer on seeing a lion.

No comment is necessary.

* दूरभ्रष्टाः स्पष्टाः श्रीषेणार्य्यभटविष्णुचन्द्रेषु ।
यस्मात् कुजादयस्ते विदुषां नैवादरस्तस्मात् ॥३॥

3. As the apparent planets beginning with Mars as derived from the works of Śrīṣeṇa, Āryabhaṭa and Viṣṇucandra are far deviated from their true places, the works of these authors are therefore not valued among the learned.

न स्फुटमार्य्यभटोक्तं स्पष्टीकरणं यतस्ततो वक्ष्ये ।
भानुमतो मन्दोच्चं राशिद्वयमंशकाश्च सप्तदश ॥४॥

4. As the process of finding the apparent places of planets as given by Āryabhaṭa does not make them agree with observation I shall therefore speak of this process. Of the sun the apogee is at two signs and seventeen degrees.

Brahmagupta here says that the longitude of the sun's apogee =2 signs 17°, whereas it is stated to be 80° or=2 signs 20° in the *Khaṇḍakhādyaka*.

According to the astronomical constants as given in the *Conn. des Temps*, the longitude of the sun's apogee in 499 A.D. (*i.e.*, 1,400 years before 1900 A.D.) was,

= 77° 19′ 19·44″ according to *Conn. des Temps*' equation,

= 76° 40′ 37·22″ according to Newcomb's equation.

The mean of these two=77° very nearly. Hence Brahmagupta's determination of the sun's apogee was more correct than that of Āryabhaṭa. The *Aryabhaṭiya* states the longitude to be 78° which also is less correct than what Brahmagupta gives here.

* *Brāhma-sphuṭa-siddhānta*, II, 47.

व्यगणात् खरुद्रगुणिताद्
भवशरयुताच्छशितिखाग्निहतात् ।
भगणादि फलं शोध्य-
मर्धमृचन्द्राच्छशाङ्कोच्चम् ॥५॥

5. Multiply the *ahargaṇa* by 110, increase the product by 511 and divide by 3031; subtract the result taken as revolutions, etc., from the mean moon; the final result is the moon's apogee.

Evidently Brahmagupta assumes that the anomalistic month $= \frac{3031}{110}$ days. This convergent to the anomalistic month was known to the author of the old *Vasiṣṭha siddhānta* as summarised in the *Pañca-siddhāntikā*.*

According to Brahmagupta the length of the anomalistic month,

$$= \frac{1582236450000 - 4320000000}{57753300000 - 488105858}\dagger \text{ days,}$$

$= 27{\cdot}55454641$ days, which is for 1900 A.D.

$= 27{\cdot}5545502$ days according to Radau,

$= 27{\cdot}554602$ days according to the *Āryabhaṭīya*.

Here also Brahmagupta is more accurate. Again the length of the sidereal period of the moon's apogee

$$= \frac{1577918450000}{488105858} \text{ days,}$$

$= 3232{\cdot}732048$ days.

Āryabhaṭa's value of the same

$= 3231{\cdot}987844$ days

The modern value of it

$= 3232{\cdot}3754$ days.

Hence Brahmagupta's result is by ·3566 of a day out, while Āryabhaṭa's is by ·3876 *da.* in.

* *Pañca-siddhāntikā*, II, 2-6.

† *Brāhma-sphuṭa-siddhānta*, I, 15, 16, 18 and 20.

पञ्चगुणा यमरामा वसुपक्षा भूयमाः शशाङ्कभुवः ।
पञ्चेत्युत्तर-मन्दकर्मणि भानोः स्फुटखण्डकाः कथिताः ॥६॥
सप्तनगाः शशिमुनयश्चन्द्ररसाः सप्तसागराः शशिनः ।
खगुणाः पुष्करचन्द्राः मान्द्याः स्युः खण्डकाश्शशिनि ॥७॥

6-7. 35′, 32′, 28′, 21′, 13′, 5′ are the tabular differences of the sun's equations in the *manda* operation as spoken of in this *uttara* portion. 77′, 71′, 61′, 47′, 30′, 10′ are the tabular differences of the moon's *manda* equation (*i.e.*, equation of apsis).

These tabular differences correspond to the tables of equations already given in Chapter I, stanzas 16 and 17; but the new method of the next stanza here teaches how correctly to calculate the sun or the moon's equation for any given value of the anomaly by using the **second difference**.

गतभोग्यखण्डकान्तरदलविकलवधात् शतैर्नवभिराप्त्या ।
तद्युतिदलं युतोनं भोग्यादूनाधिकं भोग्यम् ॥८॥

8. Multiply the residual arc left after division by 900′ (*i.e.*, by 15°), by half the difference of the tabular difference passed over and that to be passed over and divide by 900′ (*i.e.*, 15°): by the result increase or decrease, as the case may be, half the sum of the same two tabular differences; the result which, whether less or greater than the tabular difference to be passed, is the true tabular difference to be passed over.

The rule given here applies to the case of all functions hitherto considered in the *Khaṇḍakhādyaka*, which are tabulated at the difference of 15° of arc of the argument They are—

(*i*) The tabular differences of the sun's equation.

(*ii*) ,, ,, ,, ,, moon's equation.

(*iii*) ,, ,, ,, ,, "sines."

We illustrate the rule by an example belonging to the table of sines.

Illustration.—To find the 'sine' of 57°.

Brahmagupta's table of 'sines' in *Khaṇḍakhādyaka* is as follows:—*

Arc	'Sine'	Tabular Difference	Second Difference
0°	0		
15°	39	39	
30°	75	36	−3
45°	106	31	−5
60°	130	24	−7
75°	145	15	−9
90°	150	5	−10

Now $57° = 3420 = 900' \times 3 + 720'$. Thus three of the tabular differences are considered as passed over; the last one being 31 and the one to be passed over is 24.

The true tabular difference by the rule, for the arc of 57°,

$$= \frac{31+24}{2} - \frac{720}{900} \times \frac{31-24}{2}.$$

Hence the 'sine' of 57°

$$= 39+36+31+\frac{720}{900}\left(\frac{31+24}{2} - \frac{720}{900} \times \frac{31-24}{2}\right)$$

$$= 125{\cdot}76.$$

As worked out from the logarithm tables the same

$$= 125{\cdot}80.$$

Again 'sine' of 57° from Brahmagupta's formula

$$= 106 + \frac{720}{900} \times 24 + \frac{31-24}{2} \times \frac{720}{900} - \left(\frac{720}{900}\right) \times \frac{31-24}{2},$$

$$= 106 + \frac{720}{900} \times 24 + \frac{720}{900}\left(\frac{720}{900} - 1\right) \times \frac{24-31}{2}, †$$

which is the modern form of the interpolation equation up to the term containing the second difference. Brahmagupta thus takes a decidedly improved step here and is undoubtedly the first man in

* Chapter I, 30; also Chapter III, 6.

† *Cf.* Ball's Spherical Astronomy, p. 18.

the history of mathematics who has done this. The next stanza directs the corrections that have to be made to the equations of apsis of the sun and the moon as given in the *Khaṇḍakhādyaka* proper.

द्विश्रुतांशोनं रविफलमिन्दोर्वसुवेदभागयुतम् । *
अर्कफलभुक्तिघाताद् भगणकलाप्तं भुजान्तरं रविवत् ॥८॥

9. The sun's equations are to be made less by $\frac{1}{42}$nd part and the moon's equations, increased by $\frac{1}{48}$th part. Multiply the sun's equation by a planet's daily motion in minutes and divide by the number of minutes of a whole circle and this is called *Bhujāntara* correction and applied in the same way to the planet as the equation is applied to the sun.

The *Khaṇḍakhādyaka* proper applies this *Bhujāntara* due to the equation of time to the moon alone.† The first half states the corrections that are to be made to the equations of the sun and the moon. The sun's epicycle of apsis has the dimension 14° in the *Khaṇḍakhādyaka* proper.‡ This correction would make its dimension to be $= 14°(1 - \frac{1}{42}) = 13°\ 40'$.

The correction to moon's equations would make the epicycle's dimension $= 31°(1 + \frac{1}{48}) = 31°\ 38'\ 45''$.

Pṛthūdaka's commentary would make this $= 31°(1 + \frac{1}{52}) = 31°\ 35'$.

सार्धकृतेषुगुणोनादहर्गणाद् द्विनवमुनिरसैर्भक्तात् ।
यन्मण्डलादि लब्धं चक्रात् संशोध्य तत्पातः ॥१०॥

10. Deduct $354\frac{1}{2}$ from the *ahargaṇa*, divide the remainder by 6792; subtract the quotient that is obtained in revolutions, etc., from the circle: the result is the longitude of the ascending node.

* Pṛthūdaka's reading seems to be दस्रेषुभागयुतम् ।

† Chapter I, 18. *Cf.* Modern *Sūrya-siddhānta*, II, 46.

‡ Chapter I, 16, 17.

Here Brahmagupta gives the approximate period of the sidereal revolution of the moon's node to be $=6792$ days. This according to his *Brāhma-sphuṭa-siddhānta*

$$= \frac{1577916450000}{232311168} \text{ days}$$

$= 6792{\cdot}25396$ days, which according to Lockyer

$= 6793{\cdot}39108$ days, while this is

$= 6794{\cdot}75083$ days according to the *Khaṇḍakhādyaka*.

Hence Brahmagupta's attempt at correction makes the node quicker than it actually is

सप्तदशांशैरधिकं भौमस्योच्चं गुरोर्दशभिरंशैः ।
सितशीघ्रात् शतमुनयो लिप्ताः शोध्याः शनेः फलं मान्द्यम् ।
पञ्चांशोनं शैघ्रं षोडशभागाधिकं बुधस्य फलम् ॥११॥

11. Of Mars the apogee (the aphelion point) is to be increased by 17°, that of Jupiter by 10°; from the *Śīghra* of Venus 74′ are to be subtracted; Saturn's equation of apsis should be decreased by its one-fifth; the *Śīghra* equation of Mercury should be increased by one-sixteenth.

This stanza says that in 499 A.D. Mar's aphelion point had a longitude of 127°, of Jupiter the longitude of the aphelion was 170°.*

According to Newcomb's rule, the longitude of the aphelion point of Mars in 499 A.D. works out to have been $=128° \, 28' \, 12''$.

According to the *Conn. des Temps'* rule the same was $=128° \, 27' \, 51''$.

Hence Brahmagupta's determination of Mars's aphelion is correct within 1° 30′, and is therefore quite satisfactory. According to the *Khaṇḍakhādyaka* proper it was 110°, while according to the *Āryabhaṭīya* it was 118°.

Again according to this stanza Jupiter's aphelion had a longitude of 170° in 499 A.D.

According to *Conn. des Temps'* rule the same was $=170° \, 25'$.

Hence Brahmagupta is here also very accurate. According to the *Khaṇḍakhādyaka* proper it was 160°, while according to the *Āryabhaṭīya* it was 180°.

* Chapter II, 6.

The next two stanzas teach how to work out the correct *Śīghra* equation from the tabulated differences of equations for the stated intervals of the *Śīghra* anomaly as given in Chapter II.

भुक्तागतिफलांशगुणा भोग्यगतिर्भक्तागतिहृता लब्धम् ।
भुक्तागतेः फलभागास्तद्भोग्यफलान्तरार्धहतम् ॥१२॥
विकलम् भोग्यगतिहृतं लब्धेनोनाधिकं फलैक्यार्धम् ।
भोग्यफलादधिकोनं तद्भोग्यफलं स्फुटं भवति ॥१३॥

12-13. Multiply the increase of the *Śīghra* anomaly to be passed over by the increase of the *Śīghra* equation passed over and divide by the increase of the anomaly passed over; the result is the number of degrees in the adjusted increase of the equation passed over. Multiply half the difference of this result and the increase of the *Śīghra* equation to be passed over by residue (of the anomaly left after subtracting as many as possible of the preceding intervals of *Śīghra* anomaly) and divide by the increase of the *Śīghra* anomaly to be passed over: by the new result decrease or increase as the case may be, half the sum of the same two increases of the equations; the final result which is either greater or less than the increase of the *Śīghra* equation to be passed over, is the true increase of the equation to be passed over."

The meaning is made clear by an illustration.

Illustration:—To find the *Śīghra* equation of Mars for the *Śīghra* anomaly of 80°.

Mars's tabulated differences of the *Śīghra* equation are as follows:—

Increase of the *Śīghra* anomaly	Increase of the *Śīghra* equation
28°	11°
32°	12°
30°	10°
31°	7°
&c.	&c.

Here the intervals of anomaly are not equal.

The residue of the *Śīghra* anomaly

$$= 80° - (28° + 32°) = 20°.$$

The rules stated above say that for the last 30° of the *Śīghra* anomaly the change in the *Śīghra* equation,

$$= \frac{12° \times 30°}{32°} = 11°\ 15'.$$

Now as in stanza 8, the *Śīghra* equation for 80° of the *Śīghra* anomaly of Mars

$$= 11° + 12° + \frac{20}{30}\left\{\frac{11°\ 15' + 10°}{2} - \frac{20}{30}\left(\frac{11°\ 15' - 10°}{2}\right)\right\}$$

$$= 29°\ 48'\ 20''.$$

If the *Śīghra* equation were worked out by the simple rule of proportional parts it would have been

$$= 11° + 12° + \frac{20}{30} \times 10° = 29°\ 40'.$$

Now Mars's *Śīghra* epicycle having a circumference of 234°, while the deferent has a circumference 360°, the *Śīghra* equation correctly worked out for the anomaly of 80°,

$$= 29°\ 54'\ 27''$$

Thus here also Brahmagupta takes a distinctly improved step in interpolation.

चापानयने नवशतविकलवधाद् भोग्यलब्धलिप्ताभिः ।
कृत्वा खण्डकमसकृत् तल्लब्धकला विकलचापम् ॥१४॥

14. In finding the arc corresponding to a given 'sine', find the residue left after subtracting as many as possible of the tabular differences of 'sines', multiply it by 900 and divide by the tabular difference to be passed over: by means of the minutes of arc obtained find the true tabular difference by repeating the process and thus find the minutes of arc corresponding to the required residue of the 'sine'.

This stanza teaches how to find the arc correctly by using the tabular differences as described in stanza 8 of this chapter. The method of adjusted interval of the preceding stanza would have however served the purpose equally well.

This finishes the chapter IX of the Khaṇḍakhādyaka being the introductory chapter of the Uttara Khaṇḍakhādyaka, which treats of Brahmagupta's own corrections to the astronomical constants and improved methods of Interpolation.

CHAPTER X

On Conjunction of Stars and Planets.

This section treats of the conjunction of planets and the *yogatārās* or "Junction stars" The first two stanzas describe the number of stars in each lunar mansion or *nakṣatra*.

मूलाजाङ्घ्रिर्बुध्न्याश्वयुगदिति शक्राग्नीफाल्गुनीद्वितयं ।
त्वाष्ट्रगुरुवरुणार्द्रानिलपौष्णान्येकताराणि ॥१॥
ब्रह्मेन्द्रयमहरीन्दुत्रितयं षड्वह्निभुजगपित्र्याणि ।
मैत्राषाढा चतुष्कं वासवरोहिण्यौ पञ्च ॥२॥

1-2. The number of stars is two in each of the following *nakṣatras* :—Mūlā, P. Bhādrapada, U. Bhādrapada, Aświnī, Punarvasu, Viśākhā, P. Phālgunī, U. Phālgunī ; Of the *nakṣatras* Citrā, Puṣyā, Śatabhiṣā, Ārdrā, Swātī, Revatī, the number of stars is one in each case ; the *nakṣatras*, Abhijit, Jyeṣṭhā, Bharaṇī, Śravaṇā, Mṛgaśirā, have each three stars ; Kṛttikā, Aśleṣā, Maghā, have six each ; Anurādhā, P. Āṣāḍhā, U. Āṣāḍhā, four each ; Dhaniṣṭhā and Rohiṇī, five each.

स्वतारागणमध्ये यास्तारा दृश्यन्तेऽतिदीप्ततारा: ।
ध्रुवविक्षेपौ तासां कथितौ ता योगताराख्या: ॥३॥

3. Of the stars in each *nakṣatra*, those that are seen to be the brightest are the "junction stars" ; of them are given the polar longitude and the polar latitude.

अष्टनखैर्मेषे गवि रदलिप्तोनैर्गुणास्वरैर्मिथुने ।
कर्कटके गुणषोडशधृतिभि: सिंहे नवत्रिघनै: ॥४॥
कन्यायां पञ्चनखैस्तुलिनि त्र्यतिधृतिभिरलिनि सेषुकलै: ।
द्विचतुर्दशातिधृतिभिर्धनुषि शशाङ्कमनुनखतत्त्वै: ॥५॥

मकरेऽष्टनखै: कुम्भे नखषड्विंशैर्झषे मुनित्रिंशै: ।
पृथगश्विन्यादीनां ध्रुवकांशैर्योगताराणां स्थै: ॥६॥

4-6. By means of the following polar longitudes of the "junction stars" of each *nakṣatra* beginning with Aśvinī, the conjunction of planets with them should be judged :—In Meṣa by 8° and 20° ; in Vṛṣa by 7° 28′ and 19° 28′ ; in Mithuna by 3° and 7° ; in Karkaṭa by 5°, 16° and 18° ; in Siṁha by 9° and 27° ; in Kanyā by 5° and 20° ; in Tulā by 3° and 19° ; in Vṛścika by 2° 5′, 14° 5′ and 19° 5′; in Dhanu by 1°, 14°, 20° and 25° ; in Makara by 8° and 20° ; in Kumbha by 20° and 26° ; in Jhaṣa by 7° and 30°.

These stanzas are the same as 1-3 of Chapter X of our author's *Brāhmasphuṭasiddhānta*.

ध्रुवकादून: पश्चादधिक: प्राग्वक्रितेऽन्यथा योग: ।
अन्यद् ग्रहमेलकवद् ध्रुवकक्रान्तेर्भविक्षेपा: ॥७॥

7. When the planet (corrected by the *Āyana dṛkkarma*) is less than the polar longitude of a "junction star," the conjunction is yet to take place ; if greater, the conjunction has already happened ; the reverse is the rule when the planet has a retrograde motion : the rest of the calculation is similar to that for the conjunction of planets. The polar latitudes are given from the end on the ecliptic of the declination corresponding to the polar longitudes.

सौम्या दशार्कविषया याम्या: शरदशभवरसा: सौम्या: ।
खं सप्त दक्षिणा: खं सौम्या: सूर्यत्रयोदशका: ॥८॥
दक्षिणतो भवयमला: सप्तत्रिंशदुदगशका याम्या: ।
अध्यर्द्धत्रिचतुष्कार्धनवमसतंशविषयशरा: ॥९॥
सौम्या द्व्यधिका षष्टिस्त्रिंशत्षट्त्रिंशदितरलिप्ता: ।
अष्टादशोत्तरा जिनषड्विंशत्यम्बराण्यंशा: ॥१०॥

8-10. They are 10°N, 12°N, 5°N, 5°S, 10°S, 11°S, 6°N, 0°, 7°S, 0°, 12°N, 13°N, 11°S, 2°S, 37°N, 1°½S, 3°S, 4°S, 8°½S, 5°⅓S, 5°S, 62°N, 30°N, 36°N, 18′S, 24°N, 26°N, 0°.

The stanzas here numbered as 7, 8, 9 and 10 are the stanzas 4, 5, 6 and 7 of the *Brāhmasphuṭasiddhānta*, X.

प्राजेशयोगताराविक्षेपांशैः कलात्रिघनहीनैः ।
आग्नेयस्य कलानामेकोनत्रिंशता हीनैः ॥११॥
पञ्चदशकलाहीनैश्चित्रायाः सप्तभिर्विशाखायाः ।
षट्सप्तत्या मैत्रस्यैन्द्रस्य त्रिंशता हीनैः ॥१२॥

11-12. Of the junction star of Rohiṇī, the polar latitude has to be diminished by 27′, of that of Kṛttikā by 29′, of that of Citrā by 15′, of that of Viśākhā by 7′, of that of Anurādhā by 76′, of that of Jyeṣṭhā by 30′.

The above are the stanzas 8 and 9 of Chapter X of the *Brāhmasphuṭasiddhānta*.

Before we proceed any further it is necessary to put these statements in a tabular form.

Nakṣatras or Lunar mansions	No. of stars in, according to *Khaṇḍakhādyaka*	No. of stars in, according to *Śākalya-Saṁhitā*	Descriptions of *nakṣatras* according to *Śākalya-Saṁhitā*	Identification	Polar longitude of junction star			Polar latitude of junction star	Identification of "junction stars" by Burgess
					signs	dgrs.	mins.		
Aświnī	2	3	Head of a horse.	α, β and γ Arietis	0s.	8°	0′	10° N	β Arietis
Bharaṇī	3	3	*Yoni*	Musca	0s.	20°	0′	12° N	35, 41 Arietis
Kṛttikā	6	6	A razor	Pleiades	1s.	7°	28′	4° 31′ N	Alcyone
Rohiṇī	5	5	A cart or an *ekkā* of the U. P.	Hyades	1s.	19°	28′	4° 33′ S	Aldebaran
Mṛgaśirā	3	3	Head of a deer.	λ, ϕ_1, and ϕ_2 Orionis	2s	3°	0′	10° S	λ Orionis
Ārdrā	1	1	A jewel.	α Orionis	2s.	7°	0′	11° S	α Orionis
Punarvasu	2	2	A house	Castor and Pollux (α and β Gemini)	3s.	3°	3′	6° N	Pollux
Puṣyā	1	3	An arrow-head	Presepe (δ, η, γ Cancri)	3s.	16°	0′	0°	δ Cancri
Aśleṣā	6	5	A wheel	ξ, ε, δ, σ, η Hydra	3s.	18°	0′	7° S	ε Hydra
Maghā	6	5	A wall	α, η, γ, ξ, μ, ε Leonis	4s.	9°	0′	0°	Regulus
P. Phālgunī	2	2	A *carpoy*	δ and θ Leonis	4s.	27°	0′	12° N	δ Leonis
U. Phālgunī	2	5	Do.	β Leonis & β Virgo	5s.	5°	0′	13° N	β Leonis
Hastā	6	5	A hand	Corvus	5s.	20°	0′	11° S	γ & δ Corvi
Citrā	1	1	A pearl	Spica	6s.	3°	0′	1° 45′ S	Spica
Swātī	1	1	A piece of coral	Arcturus or α Bootis	6s.	19°	0′	37° N	Arcturus
Viśākhā	2	2	A gate	α and β Libra	7s.	2°	5′	1° 23′ S	ε Libra
Anurādhā	4	3	An offering to the gods	ω, δ and π Scorpionis	7s.	14°	5′	1° 44′ S	δ Scorpionis
Jyeṣṭhā	3	3	An ear pendant	Antares with the two stars on its sides. (α, σ and τ Scorpionis)	7s.	19°	5′	3° 30′ S	Antares
Mūlā	2	9	The tail of a lion	ε, μ, ζ, η, θ, ι, κ, λ, υ Scorpionis	8s.	1°	0′	8° 30′ S	λ Scorpionis
P. Āṣāḍhā	4	2	The tusk of an elephant	δ, γ, λ, ε, Sagittarius	8s.	14°	0′	5° 20′ S	δ Sagittarius
U. Āṣāḍhā	4	2	A *carpoy*	σ, ζ, φ, τ Sagittarius	8s.	20°	0′	5° S	τ Sagittarius
Abhijit	3	3	An ear of an elephant	α, β and γ Lyra	8s.	25°	0′	62° N	α Lyra
Śravaṇā	3	3	A *mṛdaṅga* (tabor)	α, β and γ Aquilæ	9s.	8°	0′	30° N	α Aquilæ
Dhaniṣṭhā	5	5	Do.	α, β, γ, δ & ε Delphinis	9s.	20°	0′	36° N	α Delphinis
Śatabhiṣā	1	100	A circle	λ Aquaris & other stars	10s.	20°	0′	18′ S	λ Aquaris
P. Bhādrapada	2	2	A *carpoy*	α and β Pegasi	10s.	26°	0′	24° N	α Pegasi
U. Bhādrapada	2	2	Do	γ Pegasi and α Andromeda	11s.	7°	0′	26° N	γ Pegasi
Revatī	1	32	A *mṛdaṅga*	ζ Piscium	0s.	0°	0′	0°	ζ Piscium

About the configurations of *nakṣatras*, the "junction stars" and other details, the reader will find a good summary of the researches of Colebrooke, Bentley and Burgess in Brennand's Hindu Astronomy, pp. 36-43.*

त्रीणि ब्राह्मात् सार्पद्वितयं हस्तादित्यदैवतायाः षट् ।
एतानि दक्षिणदिशि विक्षिप्तान्यन्यानि चोत्तरतः ॥१३॥

13.* Three stars of Rohiṇī, two Aśleṣā, six of Hastā of which the presiding deity is Sun are thrown (*i.e.* lie) on the south of the ecliptic, the remaining stars of these *nakṣatras* are on the north of the ecliptic.

छादयति योगतारां मानार्द्धोनाद्भविक्षेपात् ।
स्फुटविक्षेपो यस्याधिकोनको भवति समदिक्स्थः ॥१४॥

14. A planet being on the same side of the ecliptic occults a 'junction star', if its polar latitude be either greater or less than that of the 'junction star' decreased or increased by the semidiameter of the planet.

Let the polar latitude of the "junction star" be λ_1 and that of the planet be λ_2; at the instant of occultation either

$\lambda_1 - \lambda_2 <$ semi-diameter of the planet,

or $\lambda_2 - \lambda_1 <$,, ,, ,, ,, ;

∴ either $\lambda_2 > \lambda_1$ − semi-diameter.

or $\lambda_2 < \lambda_1$ + semi-diameter.

विक्षेपांशद्वितीयादधिको वृषभस्य सप्तदशभागे ।
यस्य ग्रहस्य याम्यो भिनत्ति शकटं सरोहिण्याः ॥१५॥

15. When a planet's polar longitude is at 17° of the sign of Vṛṣa and its polar latitude is greater than 2°S, it occults the cart of Rohiṇī

विक्षेपान्ते सौम्ये तृतीयतारकं भिनत्ति पित्र्यस्य ।
इन्दुर्भिनत्ति पुष्यं पौष्णं वारुणमविक्षिप्तः ॥१६॥

* *Cf.* Alberuni's India, Vol. II, pp. 84-85.

16. When the moon has the maximum north polar latitude she occults the third star of Maghā; when she has no celestial latitude she occults Puṣyā, Revatī and Śatabhiṣā.

Bhaṭṭotpala's text being hopelessly corrupt in the remaining chapters, we give up our attempt at reconstructing and translating them.

This finishes Chapter X of the Khaṇḍakhādyaka which relates to the Conjunction of Stars and Planets, being the second chapter of the Uttara Khaṇḍakhādyaka.

APPENDIX I

Hindu Luni-solar Astronomy.

1. In the present paper it is proposed to discuss the astronomical constants and the equations in Hindu luni-solar astronomy and to present a comparative view of these quantities with the corresponding ones in Greek and modern astronomy. It will be shown that in many cases the Hindu values of these constants are more accurate than the Greek values, and in Hindu lunar astronomy the equations or inequalities discovered are the most startling.

2. *Solar Astronomy.*

In solar astronomy the length of the year was determined by Āryabhaṭa * from the heliacal risings of some bright star at the intervals of 365 and 366 days.

(1) The year according to the *Āryabhaṭīya*

$$= \frac{1577917500}{4320000} \text{ days} = 365{\cdot}2586805 \text{ days},$$

$$= 365 \; da. \; 6 \; hrs. \; 12 \; mins. \; 29{\cdot}64 \; secs.$$

(2) The same $= \frac{1577917800}{4320000}$ days $= 365{\cdot}25875$ days,

$= 365$ *da.* 6 *hrs.* 12 *mins.* 36 *secs.*, according to the *Khaṇḍakhādyaka*, the *Sūrya Siddhānta* of Varāha and the modern *Sūrya Siddhānta*.

(3) It is $= \frac{1577916450}{4320000}$ days $= 365{\cdot}2584375$ days,

$= 365$ *da.* 6 *hrs.* 12 *mins.* 9 *secs.*, according to the *Brāhma Sphuṭa Siddhānta* of Brahmagupta.

* P. C. Sengupta, "Āryabhaṭa's Method of determining the Mean Motions of Planets," *Bulletin of the Calcutta Mathematical Society*, Vol. XII, No. 3,

Now the mean sidereal year

$= 365$ *da.* 6 *hrs.* 9 *mins.* 9·3 *secs.* (*Lockyer*).

The mean anomalistic year

$= 365$ *da.* 6 *hrs.* 13 *mins.* 49·3 *secs.* (*Lockyer*).

The mean tropical year

$= 365$ *da.* 5 *hrs.* 48 *mins.* 46·054 *secs.* (*Lockyer*).

Though we take that the Hindu year was designed to be the sidereal year, it approached most closely the anomalistic year; and its excess over the sidereal year was about 3 minutes. From this consideration it appears that the Indian astronomers were justified in taking the sun's apogee to be fixed.

Against the error of +3 *min.* in the Hindu sidereal year, we may point out that—

(1) The Hipparchus-Ptolemy tropical year,

$= 365$ *da.* 14′ 48″ in sexagesimal units,*

$= 365$ *da.* 5 *hrs.* 55 *min.* 12 *secs.*, which has an error of about +6 min.

(2) Meton's sidereal year $= (365 + \frac{1}{4} + \frac{1}{76})$ days †

$= 365$ *da.* 6 *hrs.* 18 *min* 57 *secs.* which has an error of +9 *min.* 48 *secs* nearly.

(3) The Babylonian sidereal year was $4\frac{1}{2}$ *min* too long. ‡ Thus the Hindu value of it is closer to the true value.

Again in 150 A.D. the longitude of the sun's apogee according to the *Conn. des Temps* was

$$= 101° \; 13' \; 15''{\cdot}17 - 6189''{\cdot}03 \times \left(\frac{1900-150}{100}\right)$$

$$+ 1''{\cdot}63 \times \left(\frac{1900-150}{100}\right)^2,$$

$$= 71° \; 16' \; 26''{\cdot}37,$$

while Ptolemy states it to be 65°30′ § which was wrong by −5°36′27″.

* *Syntaxis*, Karl Manitius's edition, Vol. I, p. 146.

† *Ibid*, p. 145.

‡ *Encyclopædia Britannica*, History of Astronomy.

§ *Syntaxis*, Vol. I, p. 148. The *Romaka Siddhānta* of the *Pañca siddhāntikā*, VIII, 2, indicates the sun's apogee to be at longitude of 75°; this was perhaps a correction made by Lāṭadeva to the Greek constant.

In 500 A.D. (Aryabhaṭa's time), the longitude of the sun's apogee by the same rule works out to be $=77° 19' 19·44''$.

Āryabhaṭa states this to be 78° in the *Āryabhaṭīya*, Brahmagupta in the *Uttarādhyāya of the Khaṇḍakhādyaka* states the same to be 77°, while the *Khaṇḍakhādyaka* gives it as$=$80°. Hence the Hindu findings of the longitude of the sun's apogee were more accurate.

Again as to the sun's equations of the centre we find that the *Āryabhaṭīya* states the periphery of the sun's epicycle to be 13°30′. The *Khaṇḍakhādyaka* gives it as 14°; while according to the Hindu form Ptolemy's value of the same is 15°. Hence according to these writers, the sun's equations at 90° of the mean anomaly are:—

According to the *Āryabhaṭīya*$=2° 8' 54''$.
,, ,, *Khaṇḍakhādyaka*$=2° 14' 0''$.
,, ,, Brahmagupta$=2° 7' 20''$.
,, ,, Ptolemy$=2° 23' 3''$.
The modern value $=1° 55' 9''·7$.

Thus the Hindu equations of the sun are generally more correct than the Greek ones. The Hindu constants in solar astronomy are thus, generally, more accurate than the Greek ones. We now turn to the Hindu Lunar astronomy.

3. *Lunar Astronomy.*

Before discussing the constants in Hindu lunar astronomy it is necessary to state something as to the time when the moon was observed by our ancient astronomers and the astronomers from Āryabhaṭa I (499 A.D.) to Pṛthūdaka Swāmī (864 A.D.). The months were reckoned from the first visibility of the crescent at the time of the *Mahābhārata* (1400 B.C.). We have a passage in the *Bhīsmaparvan* where Vyāsa speaks of the evil omens on the eve of the Kurukṣetra war thus—

"चन्द्रसूर्य्यावुभौ ग्रस्तावेकामासीं त्रयोदशीम् ।"

"That the moon and the sun have been both eclipsed on the 13th days of the light and dark halves of the same month."

The eclipses could not take place on the 13th days of the month unless the months were reckoned from the first visibility of the crescent. This was the custom in Babylonia and it has still survived in the Mahomedan world. Even in the *Pañca-siddhāntikā* of Varāhamihira (540 A.D.) there is a special chapter on शशिदर्शनम् or the first visibility of the crescent. It is thus clear that the practice was to observe the moon when very near the sun.

Again Āryabhaṭa says that "रवीन्दुयोगात् प्रसाधितश्चेन्दुः," "the moon was determined from her conjunctions with the sun." The moon was observed by him at the time of solar eclipses, or at the time of the first visibility of the crescent.

Even up to the time of Pṛthūdaka the accuracy in lunar astronomy was chiefly aimed at at the time of eclipses. Thus in his commentary on the *Khaṇḍakhādyaka*, IV, he makes the following introductory remarks:—

"All knowledge relating to (luni-solar) astronomy is desired by the wise (or cultured) specially for knowing the right instants of opposition or conjunction; these instants are however not visible to the eye. Of other things such as *tithis*, *nakṣatras* and *karaṇas*, as the planets, the sun and the moon, are not clearly observed, their beginnings and ends are not visible. Men see the agreement between calculation and observation at the times of solar and lunar eclipses. Hence the word of the astronomer is esteemed amongst men even in respect to such things as *tithis*, etc."*

Thus the chief aim of the ancient Indian astronomers was to calculate the eclipses accurately and the moon was observed chiefly at lunar or solar eclipses, though the time for observation related also to the finding of the first visibility of the crescent. This latter phenomena did not perhaps lead them to directly observing the moon's position at such times by using instruments.

3*a*. *Moon's Mean Motion.*

The practice of observing the moon at the time of the eclipses alone led to the determination of the synodic month with the following results:—

(*i*) Mean synodic month according to the *Āryabhaṭīya*

$$= \frac{1577917500}{57753336 - 4320000} \text{ days,}$$

$$= 29·530582 \text{ days.}$$

* "बाहुल्येन पर्व्वज्ञानार्थं सकलं ज्ञानमिष्यते शिष्टैः। तेषां च पर्व्वणां दर्शनं नास्ति। अन्येषामपि तिथिनक्षत्रकरणानां तस्मात् तेषां शशिभास्करयोरव्यक्तिर्यस्मात्। शशिभास्करग्रहणयोर्दृग्गणितैक्यं लोकः पश्यति। तस्मात्तिथ्यादिष्वप्यर्थेषु दैवज्ञवाक्यं लोके आद्रियते॥"

(ii) The same according to the *Khaṇḍakhādyaka*
= 29·5305874 days,

(iii) The same according to the *Brāhma-sphuṭa-siddhānta*
= 29·530582 days.

(iv) The same according to Ptolemy
= 29 da. 31′ 50″ 8‴ 20⁗ in sexagesimal units,
= 29·5305927 days.

The modern value according to Newcomb and Radau
= 29·5305881 days.

Hence the *Khaṇḍakhādyaka* mean length appears to be the closest approximation.

The mean sidereal month must have been deduced from the mean synodic month and the year adopted. Hence no comparison need be made of this element here.

We next consider the sidereal periods, the nodes and the apogee of the moon. These are shown below:—

According to	Sid. Per. of Moon's Apogee	Sid. Per. of the Ascending Node
Āryabhaṭīya	3231·987079 da.	6794 749511 da.
Khaṇḍakhādyaka	3231·987844 da.	6794·750834 da.
Brāhma-sphuṭa-siddhānta	3232·73411 da.	6792·25396 da.
Ptolemy	3232·617656 da.	6796·45571 da.
Modern values (*Lockyer*)	3232·37543 da.	6793·89108 da.

Here also the Hindu values show a closer approximation to the true values, Brahmagupta's figures representing the nearest approach.

3b. *Other Constants.*

So far the Hindu values of the constants have been more accurate than the Greek ones; but as to the inclination of the moon's orbit the Greek value is more accurate than the Hindu value.

Inclination of the lunar orbit

Hindu value = 4° 30′.
Greek value = 5° 0′.
Modern mean value = 5° 8′ 43″·427 (*Brown*)

This discrepancy confirms the conclusion, that the observation of the moon was restricted to the time when she was near a node, either at solar or lunar eclipses, where a small error of observation magnified itself into about half a degree.

We now turn to the *parallaxes* of the sun and the moon:—

	Sun's Mean Hor. Parallax	Moon's mean Hor. Parallax
Āryabhaṭīya	3′ 55″·62	52′ 30″
Khaṇḍakhādyaka and Brahmagupta	3′ 56″·5	52′ 42″·3
Ptolemy	2′ 51″	53′ 34″
Modern values	0′ 8″·806	57′ 2″·79

As to the sun's horizontal parallax, the ancients were of course totally wrong, but in respect to that of the moon their values were fairly approximate.

We next consider the *angular semi-diameters* of the sun and the moon. These are:—

	Moon's Mean Semi-diameter	Sun's Mean Semi-diameter
Āryabhaṭīya	15′ 45″	16′ 29″·4
Khaṇḍakhādyaka (*Brāhmasphuṭa-siddhānta*)	16′ 0″·22	16′ 15″
Ptolemy	17′ 40″	15′40″
Modern values	15′ 33″·60	16′1″·8

Here also the Hindu values are more accurate than the Greek values.

3c. *Moon's Equations—The First Equation.*

It remains now to consider the moon's equations in Hindu astronomy. As has been pointed out before, observation was up to the time of Brahmagupta, restricted to the time of eclipses perhaps also of syzygies.

The modern form of the moon's equations is

$$=377' \sin(nt-a)+13' \sin 2(nt-a)+\ldots$$
$$+76' \sin\{2(nt-\odot)-(nt-a)\}+40' \sin 2(nt-\odot)+\ldots^{*}$$

where nt = mean longitude of the moon, a the longitude of the perigee, $\odot$ = longitude of the sun.

Here the first two terms, *viz.*, $377' \sin(nt-a)+13' \sin 2(nt-a)$, are due to elliptic motion about the earth in one focus; the term $76' \sin\{2(nt-\odot)-(nt-a)\}$ is known as the evection. We combine a part of the first term with the evection term and the expression for the equation of centre becomes

$$=301' \sin(nt-a)+13' \sin 2(nt-a)+\ldots+152 \sin(nt-\odot)\cos(\odot-a)$$
$$+40' \sin 2(nt-\odot).$$

Now at syzygies and eclipses $\sin(nt-\odot)$ and $\sin 2(nt-\odot)$ will very nearly vanish. Hence according to modern astronomy at the syzygies and eclipses, the chief term of the moon's equation= $301' \sin(nt-a)$.

This according to the *Āryabhaṭīya*
$=300'\ 15'' \sin(nt-a)$,

,, ,, *Khaṇḍakhādyaka*
$=296' \sin(nt-a)$

,, ,, *Uttara Khaṇḍakhādyaka*
$=301'{\cdot}7 \sin(nt-a)$,

,, ,, *Brāhmasphuṭasiddhānta*
$=293'\ 31'' \sin(nt-a)$,

,, ,, Greek astronomy
$=300'\ 15'' \sin(nt-a)$ very nearly.

Hence both the Greek and the Hindu astronomers were very near the true value of the moon's equation at the syzygies and eclipses. Godfray in his *Lunar Theory*, page 107, observes, "the hypothesis of an excentric, whose apse has a progressive motion as conceived by Hipparchus served to calculate with considerable accuracy the circumstances of eclipses; and observations of eclipses, requiring no instruments, were then the only ones which could be made with sufficient exactness to test the truth or fallacy of the supposition." We next consider the second inequality of the moon.

* The accurate values of the co-efficients appear to be 377′ 19″·06, 12′ 57″·11, 76′ 26″ and 39′·30″.

3d. Moon's Second Inequality or Equation.

In ancient times it was Ptolemy who first really found a second inequality of the moon. According to Godfray (*Lunar Theory*, p. 107) "by dint of careful comparison of observations he (Ptolemy) found that the value of this second inequality in quadrature was always proportional to that of the first in the same place, and was additive or subtractive according as the first was so; and thus, when the first inequality was at its maximum or 5°1′, the second increased it to 7° 40′ which was the case when the apse line happened to be in syzygy at the same time."

It is well known that though Ptolemy discovered the second inequality in the moon's motion he was not able to ascertain its true nature. His corrections in this case are true when at the quadrature the moon's apse line passes through the sun or it is at right angles to the line joining the earth and the sun.* In the general case his construction does not lead to the elegant form of the evection term as we know it now, nor does it lead to the nice form in which it was given by later Hindu astronomers from the time of Mañjula (or Muñjāla, 854 Śaka era=932 A.D.).

As has been already pointed out, the early Hindu astronomers from Āryabhaṭa to Brahmagupta aimed at accuracy in lunar calculation only for the eclipses and syzygies, and did not interest themselves about the moon's longitude at the quadratures. Hence this second inequality is absent in the works of these makers of Indian astronomy, as also in the Pre-Ptolemaic Greek astronomy. This points to the conclusion that in both the earlier Hindu and Greek systems of astronomy, the modes of observation of the moon were copied from an earlier system of astronomy whether Babylonian or Chaldean. Even in the *Romaka Siddhānta* of the *Pañcasiddhāntikā* there is no mention of evection.† Thus inspite of the transmission of a vague system of Greek astronomy, Hindu astronomy as developed by Āryabhaṭa and Brahmagupta must be regarded as independent and original—not only from this but also from other considerations. It sought to correct the constants as were obtained from the Babylonian and the Greek systems as has in some cases been shown already.

* Godfray's *Lunar Theory*, pp. 108-110.

† *Vide* the summary in the writer's paper "Āryabhaṭa the Father of Indian Epicyclic Astronomy." *Journal of the Department of Letters*, Vol. XVIII, Calcutta University Press.

3d I. *Mañjula's Second Equation of the Moon* (932 A.D.).

We now take up in detail Mañjula's second equation of moon. It is however necessary to say something about his first inequality. This is given in the form

$$\frac{-488 \sin (nt - a')}{96 + \frac{488}{120} \cos (nt - a')} \text{ degrees,}$$

where $n t$ stands for the moon's mean longitude, a'— that of the apogee.

Hence when $nt - a' = 90°$, the equation $= \frac{488°}{96} = 5°4' = 304'$ showing an excess of $4'$ over the modern value.

It is further necessary to modify the expression for the moon's modern form of the equation by changing a to $180° + a$, as in Hindu astronomy anomaly is measured not from the perigee but from the apogee.

The modified form is

$$= -301' \sin (nt - a) + 13 \sin 2(nt - a) \ldots \quad \ldots \quad \ldots$$
$$- 152' \sin (nt - \odot) \cos (\odot - a) + 40' \sin 2 (nt - \odot) + \ldots$$

Mañjula's lines giving the second equation are—

ग्रन्दूच्चोनार्ककोटिघ्ना गत्यंशा विभवा विधोः ।
गुणो व्यर्केन्दुदोः कोट्योरूपपञ्चाप्तयोः क्रमात् ॥११॥
फले शशाङ्क-तद्गत्योर्लिप्ताद्ये स्वर्णयोर्वधे ।
ऋणं चन्द्रे धनं भुक्तौ स्वर्णसाम्यवधेऽन्यथा ॥१२॥

This may be translated as follows:—

"The (mean) daily motion of the moon diminished by 11° and multiplied by the "cosine" of the longitude of the sun diminished by that of the moon's apogee is the multiplier of the "sine" and the "cosine" of the longitude of the moon diminished by that of the sun, divided severally by 1 and 5. The results taken as minutes are to be applied negatively and positively to the moon and to her daily motion if the quantities multiplied together are of opposite signs and in the reverse order if they are of the same sign."

As to the positive or negative character of the "sine" and the "cosine" he gives the rule—

षड्भः स्वोच्चोनितः केन्द्रं षड्भूर्ध्वादिदलैर्भुजः ।
धनर्णः पदशः कोटिर्धनर्णर्णधनात्मिका ॥

"The mean planet diminished by its *ucca*, the apogee, aphelion or the *Sighra*, is called *kendra* or mean anomaly; its "sine" from above six signs (180°) arises from half circles and are respectively positive or negative and its "cosine" in different quadrants are respectively positive, negative, negative, and positive."

The convention followed is that the "sine" is negative from 0° to 180° and positive from 180° to 360° of the arc and that the cosine is positive between 0 and 90°, negative between 90° and 270° and positive between 270° and 360°.

We may now symbolically express Mañjula's second inequality thus:—

$$-(13° \; 10' \; 35'' - 11°) \times 8^{p}8' \cos (\odot - a) \times 8^{p}8' \sin (☽ - \odot),$$

where ☽ stands for the moon as corrected by the 1st equation; we leave out the correction to the moon's daily motion as given in the stanzas quoted above.

The moon's new equation comes out to be

$$= -143' \; 58'' \cos (\odot - a) \sin (☽ - \odot).$$

This, it will be seen, is exactly the modern form of the evection as combined with a part of the equation of apsis shown before. The difference in the main is that Mañjula's constant is 144', a quantity less by 8'. **In form the equation is most perfect, it is far superior to Ptolemy's, it is above all praise.** It is from this inequality, we trust, that Mañjula should have an abiding place in the history of astronomy. The next writer who gives the second equation is Śrīpati (1028 A.D.).

3d II. *Śrīpati's Second Inequality of the Moon* (1028 A.D.).

The following stanzas from Śrīpati's *Siddhānta Sekhara*, were very kindly communicated to me by Pandit Babua Misra. Though they are probably not very correct still the general meaning is clear. They are the following:—

विभविरहितचन्द्रोच्चोनभास्वद्भुजज्या
गगननृपविनिघ्नी भवयज्या विभक्ता ।
भवति चरफलाख्यं तत्पृथक्स्थं शरघ्नं
धृतमुडुपतिकर्णत्रिज्ययोरन्तरेण ॥
परमफलमवाप्तं तद्धनर्णं पृथक्स्थे ।
तुहिनकिरणकर्णे त्रिज्यकोनाधिकेऽथ ।
स्फुटदिनकरहीनादिन्दुतो या भुजज्या
स्फुटपरमफलघ्नी भाजिता त्रिज्ययाप्तम् ॥

शशिनि चरफलाख्यं सूर्यहीनेन्दुगोलात्
तदृणसुतधनं चेन्दूच्चहीनार्कगोलम् ।
यदि भवति हि साम्यं व्यस्तमेतद् विधेयम्
स्फुटगणितदृगैक्यं कर्त्तुमिच्छद्भिरत्र ॥

The passage may be translated thus:—

"From the moon's apogee subtract 90°, diminish the sun by the remainder left; take the "sine" of the result; multiply it by 160′ and divide by the radius; the result is called *caraphala*. Put it down in another place, multiply it by *sara* (*i.e.*, R vers $(\text{☽} - a)$, or versed sine of the moon's distance from the apogee) and divide by the difference between the moon's distance (hypotenuse) and the radius; the result is called *parama* (*cara*) *phala*, which is to be considered positive or negative according as the hypotenuse put down in another place is less or greater than the radius. Multiply the "sine" of the moon which has been diminished by the apparent sun, by the apparent *paramaphala* and divide by the radius; the final result is to be called *caraphala* to be applied to moon negatively or positively as the moon minus the sun and the sun minus the moon's apogee (diminished by 90°) be of opposite signs; if these latter quantities be of the same sign the new equation should be applied in the inverse order by those who want to make the calculation of the apparent moon agree with observation."

Symbolically—

$$\frac{160' \text{ R sin } \{\odot - (a - 90°)\}}{\text{R}} = caraphala,$$

$$\mp \frac{160' \text{ R sin } \{\odot - (a - 90)°\}}{\text{R}} \times \frac{\text{R vers } (\text{☽} - a)}{\text{H} - \text{R}} =$$

paramaphala, according as H > or < R.

The new equation

$$= \mp \frac{\text{R sin } (\text{☽} - \odot)}{\text{R}} \times paramaphala$$

$$= \mp \frac{160' \text{ R sin } \{\odot - (a - 90°)\} \text{ R vers } (\text{☽} - a) \times \text{ R sin } (\text{☽} - \odot)}{\text{R(H} - \text{R)} \times \text{R}}$$

$$= \mp \frac{160' \text{ R cos } (\odot - a) \times \text{R sin } (\text{☽} - \odot)}{\text{R} \times \text{R}} \times \frac{\text{R vers } (\text{☽} - a)^*}{\text{H} - \text{R}}.$$

* There is some uncertainty about this new fraction introduced by Śrīpati.

This equation is a slightly modified one but practically the same in form as that of Mañjula, except that the constant here is 160′, greater than his by 16′. The constant is 160′ also in Candraśekhara's form as we shall see later on. We next consider the Moon's inequalities as given by Bhāskara II in his *Bījopanaya*,* a special work on these inequalities composed in the *Saka* year 1074 (=1152 A.D.) two years after he had composed the *Siddhānta Śiromaṇi*.

3*d* III. *Bhāskara II on Moon's Inequalities* (1152 A.D.).

His preliminary statement runs thus:—

लिप्ता विधोरर्कमहीमिता मे दृग्गोचराः प्रत्यहमीक्षितस्य ।
कदम्बगोलगत-सूत्रपाते क्रान्तौ धनर्णत्वजुषो भमध्यात् ॥

Bījopanaya, St. 8.

"112′ positive or negative representing the maximum difference, have been found by me in the daily observed moon (as calculated and as observed) at that point of the ecliptic where the arc from the *kadamba* (*i.e.*, its pole) passing through the zenith cuts it."

Thus for observing the moon he selected the nonagesimal as the suitable point where the uncertainty about her parallax is zero, and found $\pm$ 112′ of arc to be the maximum difference between her calculated and observed places.

Mallabhaṭṭa, perhaps a contemporary of Bhāskara II, ascribed this difference to a supposed *Śīghrocca* of the moon. Bhāskara in stanzas 9-13, refutes the existence of the *Śīghra* in the case of the moon, the substance of his argument being (*i*) that it is against the teaching of the *Sūryasiddhānta* and other accepted authorities, (*ii*) that there is no variation of the apparent angular diameter of the moon corresponding to this alleged *Śīghra*, and (*iii*) that planets having a *Śīghra* have retrograde motion which is never the case with the moon.

The reason for his new equations are stated as follows:—

तुङ्गादाद्यपदान्तस्थाद् विधोरर्के पदार्द्धतः ।
परमं चन्द्रवैषम्यम् ऋणत्वेन समीक्ष्यते ॥२०॥
तत्तृतीयपदान्तस्थात् पृष्ठगेऽर्के पदार्द्धतः ।
परमं चन्द्रवैषम्यं धनत्वेन समीक्ष्यते ॥२१॥

* Published by the Punjab Sanskrit Book Depot, Lahore, 1926.

चन्द्रतुङ्गे च नीचे च शशाङ्कार्कग्रहौ यदि ।
मन्दस्फुटगतश्चन्द्रो निर्बीजतुल्यमीक्ष्यते ॥२२॥
ओजान्तयोर्विधोस्तुङ्गाच्छशाङ्कार्कग्रहौ यदि ।
चतुस्त्रिंशत्कलाहीनं वैषम्यं तु समीक्ष्यते ॥२३॥
अग्रतः पृष्ठतो वाऽपि रवेश्चन्द्रे पदार्धगे ।
तुङ्गतुल्ये चतुस्त्रिंशत्कलावैषम्यमीक्ष्यते ॥२४॥
एवं तन्नीचतुल्येऽपि वैषम्यं तावदेव हि ।
एवं व्यासात् समासाच्च पौनःपुन्येन वेधनात् ।
चरबीजमिदं क्लृप्तं मया सद्भिः समीक्ष्यताम् ॥२५॥

"When the moon is situated at a quadrant ahead of the apogee and with the sun at half a quadrant ahead of her, the maximum discrepancy (of 112′) is seen in the negative character.

"When the moon is situated at three quadrants ahead of the apogee and with the sun at half a quadrant behind her, the maximum discrepancy (of 112′) is seen in the positive character.

"When the eclipses of the sun and the moon take place at the apogee or the perigee of the moon the moon as corrected by the equation apsis is seen to be without any new correction called *bīja*.

"When the eclipses of the sun and the moon take place at the ends of the odd quadrants of the moon's anomaly (measured from the apogee), the discrepancy is seen to be less by 34′.

"When the moon is at the apogee, whether the sun be ahead or behind her by half a quadrant, the discrepancy amounts to be 34′.

"The same discrepancy of 34′ is observed when the moon is at the perigee and the sun is ahead or behind her by the same distance.

"Thus by analysis and synthesis, and by repeated observations, this variable correction has been devised by me: let it be seriously considered by the learned."

Bhāskara here speaks of six cases and we consider them one after another:—

The moon's equations as modified to suit *siddhāntas* are given by

$$-301' \sin (nt-a)+13' \sin 2(nt-a)\ldots\ldots$$

$$-152' \sin (nt-\odot) \cos (\odot-a)+40' \sin 2(nt-\odot)+\ldots$$

According to Bhāskara's *Siddhānta Siromaṇi*, the moon's equation of apsis

$$=-\frac{31^\circ\ 36'}{360^\circ}\times 3438' \sin (nt-a)$$

$$=-301'\ 46''\cdot 8 \sin (nt-a)$$

this agrees well with the corresponding term of the modern equation. As Bhāskara takes in all the six cases, $nt-a=90^\circ$, 270°, 0° or 180°, the second term of the equation of apsis vanishes.

Case I.

$$nt-a=90^\circ,\ nt-\odot=-45^\circ,\ \odot-a=135^\circ$$

Here the total equation of the moon

$$=-301'-(76'+40')=-301'-116'$$

this fairly agrees with Bhāskara's observation, the difference being only of 4′.

Case II.

$$nt-a=270^\circ,\ nt-\odot=45^\circ,\ \odot-a=225^\circ;$$

the total equation of the moon

$$=301'+76'+40'=301'+116',$$

this also agrees with Bhāskara's observation.

Case III,

$$nt-a=0^\circ \text{ or } 180^\circ,\ nt-\odot=0^\circ \text{ or } 180^\circ,\ \odot-a=0 \text{ or } 180^\circ,$$

the total equation $=0'$, this also agrees with Bhāskara's observation.

Case IV.

$$nt-a=90^\circ \text{ or } 270^\circ,\ nt-\odot=0^\circ \text{ or } 180^\circ,\ \odot-a=90^\circ \text{ or } 270^\circ,$$

the total equation $=\mp 301'$. This does not agree with Bhāskara's statement that the total equation

$$=\mp(301'\pm 78').$$

Case V.

$$nt-a=0,\ nt-\odot=\pm 45^\circ,\ \odot-a=\pm 45$$

the total equation

$$=0'-76'+40'=-36' \text{ or } 0'+76'-40'=+36',$$

this fairly agrees with Bhāskara's observation.

Case VI.

$$nt-a=180^\circ,\ nt-\odot=\pm 45^\circ,\ \odot-\bar{a}=180^\circ\mp 45^\circ,$$

the total equation

$$=0'+76'+40'=0'+116',\text{ or } 0'-76'+40'=0'-36',$$

this does not agree with Bhāskara's statement.

Bhāskara then states his first system of 24 equations corresponding to 24 sines in a quadrant to be

6′, 13′, 21′, 27′, 33′, 39′, 45′, 51′, 56′, 61′, 65′, 68′, 70′, 72′, 74′, 75′, 75′, 76′, 76′, 77′, 77′, 78′, 78′, 78′.*

* *Bijopanaya*, 26-28.

These equations he says—

* * * फलं ऋणे ऋणं ।

धने धं मन्दफलेन संयुतम् ॥२८॥

"are negatively added to the equation of apsis when that is negative and positively added to the same when that is positive." In other words his new equations are complements of the equation of apsis, the two together being represented by

$$-301' 46''\cdot 8 \sin (nt-a) - 78' \sin (nt-a)$$

i.e., by $-379' 46''\cdot 8 \sin (nt-a)$.

Hence next states his second set of equations depending on ☉ − ☽, to be

6′, 9′, 13′, 17′, 22′, 24′, 27′, 30′, 32′, 33′, 34′, 34′, 34′, 33′, 31′, 29′, 26′, 24′, 20′, 16′, 11′, 8′, 3′, 0′,*

and says

एताः कला ओजपदे ऋणं स्यु

र्धनं तदन्यत्र भवन्ति भूयः ।

"These minutes are *negative* in the odd quadrants of the argument and are *positive* in other quadrants."

When the value of the argument is 15° the equation is 17′,

,,	,,	,,	45°	,,	34′,
,,	,,	,,	90°	,,	0′.

Hence the new equation $= -34' \sin 2(☉ - ☽)$,

$= 34' \sin 2(☽ - ☉)$.

Here the symbol ☽ stands for the moon as corrected by the Hindu equation of apsis and its complement as given by Bhāskara. It is readily seen that Bhāskara is the first of all the Hindu astronomers to detect the equation known as "Variation." His constant, 34′, is less than the modern value by about 6′, and cannot be considered as a serious error.

We now see that the sum-total of the moon's equation as given by Bhāskara

$$= -379' 46''\cdot 8 \sin (nt-a) + 34' \sin 2(☽ - ☉),$$

the evection term being totally absent. This is a serious defect, and Bhāskara's new equations would make the moon generally more incorrect at the syzygies and eclipses than what the old Hindu equation of apsis would do.

* *Ibid*, 29-32.

Perhaps late in life when he was 69 years old in 1105 of Śaka era (=1183 A.D.) he discovered the inapplicability of his new equations at the times of eclipses and in his *Karaṇa-kutūhala* he altogether omitted these new equations which he had given in his Bījopanaya.

As to Bhāskara's second inequality which is really the complement of the equation of apsis without the evection term, it is far inferior to that of Mañjula and of Śrīpati; as we have seen their form of the second inequality combines the complement of the equation of apsis and evection in the mathematically correct form. For the discovery of such a form of the equation as of these authors, very *patient careful and frequent observation must have been coupled with very careful and nice comparison of observed facts.*

As to "variation" it was first discovered by Abul-Wefa in 976 A.D.,* which was quite forgotten when Tycho-Brahe re-discovered it in 1580 A.D. Hence Bhāskara, in 1152 A.D., re-discovered it in India 4 centuries before Tycho.

3d IV. *Candraśekhara of Orissa on the Moon's Inequalities.*

In connection with lunar inequalities it is necessary here to record what were the equations discovered or verified by M. M. Candraśekhara Siṃha of Orissa in the later half of the last century. He was educated in the orthodox Sanskrit fashion and had no acquaintance with English education. His work *Siddhānta-darpaṇa* was edited by Prof. Jogeschandra Ray, late of the Cuttack College, in 1899. Candraśekhara in his work gives four equations of the moon which are:—

(1) The equation of apsis.

(2) The Tungāntara equation or the complement of the equation of apsis in combination with evection.

(3) The fortnightly equation or variation.

(4) The *Digaṃśa* equation or the annual equation (*i.e.*, $\frac{1}{10}$ of the sun's equation).

(1) The first equation is of the form

$$-\frac{\{31°30' - 30' \cos (nt-a)\}3438 \times \sin (nt-a)}{360°} \dagger$$

$$= -300'49''\cdot 5 \sin (nt-a) + 4'46''\cdot 5 \sin (nt-a) \cos (nt-a)$$

$$= -300'49''\cdot 5 \{\sin (nt-a) + 2'23''\cdot 25 \sin 2(nt-a).$$

* Godfray's *Lunar Theory*, p. 114.

† *Siddhānta-darpaṇa*, V, 100-114.

It is seen that Candraśekhara wanted to correct the equation of apsis to the second order of small quantities as in all the Hindu authors from Brahmagupta, but Candraśekhara's form is correct though his constant is wrong.

(2) His second equation is of the form

$$\frac{160' \times 3438 \sin(a - \odot + 90^\circ)}{3438} \times \frac{3438 \sin(\text{☽} - \odot)}{3438}$$

$$\times \frac{\text{moon's appt. daily motion}^{*}}{\text{moon's mean motion}},$$

$$= -160' \cos(\odot - a) \sin(\text{☽} - \odot) \times \frac{\text{moon's appt. daily motion}}{\text{moon's daily mean motion}}.$$

Here the constant is the same as that of Śrīpati discussed before. The symbol ☽ means the moon as corrected by the equation of apsis. It is readily seen that the constant of the first term of the equation of apsis is increased by 80′, and that the constant of evection is taken at 80′. In both the cases the error is about +4′.

(3) Candraśekhara's third equation or Variation

$$= \frac{3438' \sin 2(\text{☽}_1 - \odot)}{90} = 38'\,12'' \sin 2(\text{☽}_1 - \odot),^{\dagger}$$

where ☽_1 means the moon as corrected by the 1st and 2nd equations. Here the constant is wrong by −1′ 18″.

(4) His fourth equation or the annual equation

$$= \pm \frac{1}{10} \text{ of the sun's equation of apsis,}^{\ddagger}$$

$$= \pm \frac{1}{10} \times \frac{12 \times 3438}{360} \sin(\text{sun's distance from the apogee}),$$

$$= \pm 11'\,27'\!\cdot\!6 \sin(\text{sun's distance from the apogee}).$$

The modern value of the constant is 11′ 10″. Tycho found it to be 4′ 30″, Horrocks' (1639) co-efficient was 11′ 51″.

As Candraśekhara was aware of Bhāskara's *Bījopanaya*, as also of the work of Śrīpati, his merit here lies in the discovery of the annual equation, and correction to the constant of variation.

* *Ibid*, VI, 7-9.
† *Siddhānta-darpaṇa*, VI, 11-12.
‡ *Ibid*, VI, 13.

4. *Conclusion*.

We have now come to the end of the paper. Perhaps it has been established that so far as the luni-solar astronomy is concerned Hindu astronomy is independent of Greek astronomy in respect of astronomical constants, that Hindu astronomy is generally more accurate than Greek astronomy and that Hindu astronomers were not mere "calculators."* There were observers who verified and corrected the old astronomical constants as they came down from Āryabhaṭa and Brahmagupta, who also found independently all the principal equations of the moon.

* G. R. Kaye, *Hindu Astronomy*, p. 60.

APPENDIX II

Greek and Hindu Methods in Spherical Astronomy

1. *The Aim of the Paper.*

The aim of this paper is to present a comparative view of the Greek and Hindu methods in Spherical Astronomy and to bring out the independence of the Hindu Astronomers in this subject. This comparison has not yet been properly made in the works of any European researcher from Colebrooke and Bentley early in the 19th century to Kaye in the present century. Nor have the Hindu methods in Spherical Astronomy been yet properly described in the writings of Burgess, Wilkinson, Bāpudev Śāstri and Thibaut. Hence we find Kaye writing in J.A.S.B. for 1919, No. 3 :—

"The methods by which (the rules) were obtained are buried in obscurity." Braunmühl* has stated "that the Indians were the first to utilise the method of projection in the Analemma of Ptolemy." It is intended to present the Hindu methods as clearly as possible and to show that Braunmühl has not done sufficient justice to the Indian astronomers.

As to Kaye, we shall show that his remark quoted above is due to the fact that he had to rely mostly on the English translation of the *Sūryasiddhānta* of Burgess, that with his scanty knowledge of Sanskrit could not study the works of Bhāskara II (1150 A.D.), who was the first to explain the Hindu methods clearly.

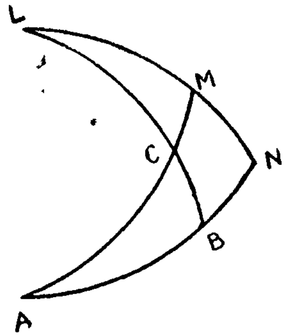
2. *Greek Methods in Spherical Astronomy.*

Of the Greek methods in Spherical Astronomy, the history begins with elementary principles only from Euclid (300 B.C.). Even in

* Heath, Greek Mathematics, Vol. II, p. 291.
Braunmühl, Geschichte der Trigonometrie, pp. 38-42.

Theodosius' Sphaerica* (about 153 B.C.) "there is nothing that can be called trigonometrical." Heath again says, "the early spheric did not deal with the geometry of the sphere as such, still less did it contain anything of the nature of the spherical trigonometry. (This deficiency was afterwards made good by Menelaus's Sphaerica.)"† Hence the Greek spherical trigonometry began with Menelaus (90 A.D.). His theorem in geometry is well-known—"If the sides of a plane triangle be cut by a transversal into six segments, the continued product of any three alternate segments is equal to the continued product of the remaining three." From this proposition he deduced the so-called "*regula sex quantitatum*" or the theorem, "if the sides of a spherical triangle be cut by an arc of a great circle into six segments, the continued product of the chords of the doubles of any three alternate segments is equal to the continued product of the chords of doubles of the remaining three segments." In plane geometry if the sides BC, CA, AB of a triangle be cut by any transversal at L, M, N, respectively, then we have

$$\frac{BL}{LC}\cdot\frac{CM}{MA}\cdot\frac{AN}{NB}=1.$$



In spherics the theorem is :

$$\frac{\text{Chord } 2\,BL}{\text{Chord } 2\,LC}\times\frac{\text{Chord } 2\,CM}{\text{Chord } 2\,MA}\times\frac{\text{Chord } 2\,AN}{\text{Chord } 2\,NB}.$$

Both these theorems are proved in Ptolemy's Syntaxis (Karl Manitius's edition, Vol. I, pp. 45-51).

If R be the radius of the sphere on which the spherical triangle ABC is constructed, then the chord of the arc 2 BL = 2 R sin BL. Hence Menelaus's theorem in spherics may be expressed as follows :

$$\frac{\sin BL}{\sin LC}\times\frac{\sin CM}{\sin MA}\times\frac{\sin AN}{\sin NB}=1$$

This theorem is true for any spherical triangle.

* Heath, Greek Mathematics, Vol. II, p. 250.

† A. A. Björnbo, 'Studien über Menelaos' Sphärik' in Abhandlungen Zur Geschichte der Mathematischen Wissenschaften for 1902, pp. 89 *et seq.*
Also, Heath, Greek Mathematics, Vol. II, pp. 261-73.

If $\angle B = AN = AM = 90°$ and L the pole of AB, then LMN is a secondary to the arc AB. There are four arcs of great circles; taking any three as forming a spherical triangle and the fourth as the transversal we readily get for the right-angled triangle ABC, the relations:—

(*i*) $\sin a = \sin b \sin A$

(*ii*) $\sin c = \tan a \cot A$

(*iii*) $\cos b = \cos a \cos c$

(*iv*) $\tan c = \tan b \cos A$

The above are some of Napier's rules for a right-angled spherical triangle, deducible from Menelaus's theorem.* They are generally sufficient in the case of such triangles. In any spherical triangle however, this theorem of Menelaus does not in any single step lead to any of the equivalents of the time-altitude or altazimuth equations in spherical astronomy. The Hindu methods, though none of them are so highly finished as Menelaus's theorem, yet are not less powerful in tackling the problems that arise in astronomy in connection with the apparent diurnal motion of the heavens. The Greek or Ptolemaic method presents no further points of interest except in its application. We now proceed to illustrate the Hindu methods and shall refer to the Ptolemaic method as occasion arises.

3. *Hindu Methods in Spherical Astronomy.*

In the Hindu methods there is no general rule to follow. It is by properties of similar right-angled triangles that a fairly complete set of accurate formulæ are obtained. These right-angled plane triangles are classified under the names,—'*Krāntikṣetras*' (triangles of declination) and '*Akṣa-kṣetras*' (triangles of latitude). We consider the following problems:—

Problem I.—To find the time of rising on the equator of a length l, of arc of the ecliptic measured from the first point of Aries.

Let ω be the obliquity of the ecliptic and R.A. the right ascension

* Three more can be deduced similarly, namely,

(*v*) $\sin c = \sin b \sin C$

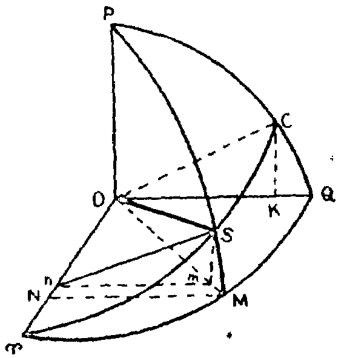
(*vi*) $\sin a = \tan c \cot C$

(*vii*) $\tan a = \cos C \tan b$.

corresponding to the longitude l, and δ, the corresponding declination. The Indian form of the equation is

$$^{*}R \sin R.A. = \frac{R \sin l \times R \sin \omega}{R \cos \delta},$$ were R is the radius of the sphere.

Note.—If R be the radius of the circle of reference, the Indian trigonometrical functions for the arc θ, are (1) the 'sine,' (2) the 'cosine' and (3) the 'versed sine.' They are respectively equal to $R \sin \theta$, $R \cos \theta$ and $R \text{ vers } \theta$.



In the adjoining figure, O is the centre of the armillary sphere, γQ, γC are quadrants of the equator and the ecliptic, respectively. P is the celestial pole, PCQ the summer solstitial colure. Join $O\gamma$, OQ, OP and OC.

Let γS be $= l$, $\gamma M = R.A.$, $CQ = \angle S\gamma M = \omega$, $SM = \delta$.

Join OS, OM. PSM is the secondary to the equator.

From C draw CK perpendicular to OQ. From S draw Sm and Sn perpendicular to OM and $O\gamma$, respectively. Join mn and from M draw MN perpendicular to $O\gamma$.

Then the triangles Smn and CKO are similar. They are called '*Krānti-kṣetras*'† or declination triangles—similar right-angled triangles having one acute angle $= \omega$.

$$\therefore \quad Sm : Sn = CK : OC$$

$$\text{or} \quad R \sin \delta : R \sin l = R \sin \omega : R$$

$$\therefore \quad R \sin \delta = \frac{R \sin l . \; R \sin \omega}{R} \quad \ldots (1)$$

* *Āryabhaṭīya*, *Gola*, 25. Varāha-Mihira in the *Pañcasiddhāntikā* (IV, 92) states it in the form $2R \frac{\sqrt{(R^2 \text{Sin}^2 l) - R^2 \text{Sin}^2 \delta}}{2R \cos \delta} = R \sin R.A.$, which is evident from the figure. Brahmagupta's equation is identical with that of Āryabhaṭa. B. S. III, 15, *Sūryasiddhānta*, III, 40-41. Also Bhāskara II, *Grahagaṇita*, Ch. VIII, St. 54-55 is in agreement with Varāha-Mihira's form.

† Bhāskara, II, *Golādhyāya*, VIII, 43-44.

Greek Method:

In the same figure* let PSC be the triangle and γMQ be the transversal. Then Menelaus's theorem gives

$$\frac{\sin PM}{\sin MS}\times\frac{\sin S\gamma}{\sin \gamma C}\times\frac{\sin CQ}{\sin QP}=1$$

$$or \quad \frac{1}{\sin\delta}\times\frac{\sin l}{1}\times\frac{\sin\omega}{1}=1$$

$$or \quad \sin\delta=\sin l.\ \sin\omega.$$

Hindu Method:—

Again by the Hindu method from the same two similar triangles we get

$$mn\ :\ n\text{S}=\text{OK}\ :\ \text{OC}$$

$$or,\quad mn\ :\ \text{R}\sin l=\text{R}\cos\omega\ :\ \text{R}$$

$$\therefore\quad mn=\frac{\text{R}\sin l\times\text{R}\cos\omega}{\text{R}}$$

Again MN : mn = OM : Om

i.e., $\quad$ R sin R.A. : mn = R : R cos δ

$$\therefore\quad \text{R}\sin\text{R.A.}=\frac{\text{R}\sin l\times\text{R}\cos\omega}{\text{R}\cos\delta}\qquad\ldots\quad(2)$$

Greek Method:

Take† PQM for the triangle and γSC for the transversal. Then,

$$\frac{\sin PC}{\sin CQ}\times\frac{\sin Q\gamma}{\sin\gamma M}\times\frac{\sin MS}{\sin SP}=1$$

$$or\quad \frac{\cos\omega}{\sin\omega}\times\frac{1}{\sin\text{R.A.}}\times\frac{\sin\delta}{\cos\delta}=1$$

$$or\quad \sin\text{R.A.}=\tan\delta\cot\omega.$$

The Hindu form of the equation is different from that of Ptolemy's. It is also better for the purpose of calculation.

Note:—From the same two similar triangles we have

$$On\ :\ \text{ON}=\text{R}\cos\delta\ :\ \text{R}$$

$$\therefore\quad On=\text{R}\cos l=\frac{\text{R}\cos\text{R.A.}\times\text{R}\cos\delta}{\text{R}}\qquad\ldots\quad(3)$$

* Manitius's edition of the *Syntaxis*, Vol. I, pp. 51-53.

† *Ibid*, pp. 55-56.

$$\text{Again, }\tan\text{R.A.}=\frac{mn}{on}$$

$$=\frac{\text{R}\sin l\times\text{R}\cos\omega}{\text{R}\times\text{R}\cos l}\qquad\ldots\quad(4)$$

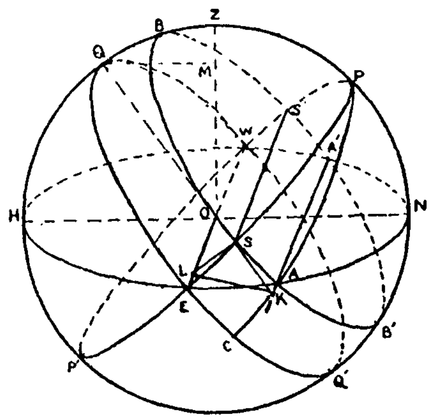
Again, mn : Sm = OK : KC

$$or\quad mn=\frac{\text{R}\sin\delta\times\text{R}\cos\omega}{\text{R}\sin\omega}$$

$$\therefore\quad \text{R}\sin\text{R.A.}=\frac{\text{MN}}{mn}\times mn=\frac{\text{R}}{\text{R}\cos\delta}\times\frac{\text{R}\sin\delta\times\text{R}\cos\omega}{\text{R}\sin\omega}\qquad\ldots\quad(5)$$

Problem II.—The problem discussed above provides the method of finding the sidereal time-intervals in which the signs of the zodiac rise on the equator. To find the corresponding times at any latitude ϕ it is necessary to calculate and apply what is the ascensional difference due to the elevation of the celestial pole. This ascensional difference is called '*carakāla*' or the variation in the length of half the day. The 'sine' of this '*carakāla*' is called '*carajyā.*' If *ch* denotes this '*carakāla*,'

$$\text{then,* R}\sin ch=\frac{\text{R}\sin\phi\times\text{R}\sin\delta\times\text{R}}{\text{R}\cos\phi\times\text{R}\cos\delta}.$$



Just as in the solution of the previous problem, the declinational triangles or '*Krānti kṣetras*' were constructed and used, so in the solution of this and other problems, another set of similar triangles were conceived and constructed and were given the name, '*Akṣa-kṣetras.*' †

Let NPZH be the meridian, NOH the north-south line passing through the observer O, P the celestial pole, OQ the trace of the equator on the meridian plane, Z the zenith. Join OZ. From Q

* *Āryabhaṭīya*, *Gola*, 26; *Pañca-siddhāntikā*, IV, 34; *Brāhmasphuṭasiddhānta*, II, 57-58; *Sūryasiddhānta*, II, 91; *Grahagaṇita*, VIII, 48-49.

† Bhāskara, *Golādhyāya* (Wilkinson and Bāpudev Sāstrī's tr.) pp. 173-76; also, Bhāskara, *Grahagaṇita*, Ch. IX, 13-17.

draw QM perpendicular to OZ. Then the triangle QOM is an '*Akṣa-kṣetra*' or a latitudinal right-angled triangle, as $\angle QOM = \phi$, the latitude of the station. Another '*Akṣakṣetra*' is thus conceived, in the same figure, let P, P′ be the north and south celestial poles, N the north point, ABA′B′ the diurnal circle of a heavenly body with declination δ, NEHW the horizon, PEP′W the six o'clock circle. Here AA′ the line of intersection of the diurnal circle with the horizon is called the "*udayāsta-sūtra*" * (or the *thread* joining the rising and setting points). SS′ the line of intersection of the diurnal circle and the six o'clock circle, is the horizontal diameter of the diurnal circle. From S draw SK and SL perpendiculars respectively to AA′ and EW. Join KL.

Now since $PN = \phi$, the latitude of the station, in the small right-angled triangle KLS, the $\angle KLS$ is also $= \phi$

$$\therefore \quad SK : SL = QM : MO$$

$$\text{or} \quad SK = \frac{SL \times QM}{MO} = \frac{R \sin \delta \times R \sin \phi}{R \cos \phi}.$$

Now SK † is a "sine" in the small circle ABA′B′ of which the radius is $R \cos \delta$; this "sine" reduced to the equator (radius R) is the 'sine' of '*cara*.'

$$\therefore \quad R \sin ch = R \sin EPA$$

$$= \frac{R \sin \delta \times R \sin \phi \times R}{R \cos \phi \times R \cos \delta}.$$

Greek Method:

Let ‡ the arc PA be produced to meet the equator at C. Take PCQ′ for the triangle and EAN for the transversal. Then we get,

$$\frac{\sin PA}{\sin AC} \times \frac{\sin CE}{\sin EQ'} \times \frac{\sin Q'N}{\sin NP} = 1$$

$$\text{or} \quad \frac{\cos \delta}{\sin \delta} \times \frac{\sin CE}{1} \times \frac{\cos \phi}{\sin \phi} = 1$$

$$\therefore \quad \sin CE = \sin ch = \frac{\sin \phi \times \sin \delta}{\cos \phi \times \cos \delta}.$$

* Bhāskara, *Gola*, VII, 39.

† This is called by the name '*kujyā*' or '*kṣitijyā*,' *i.e.*, earth-sine. Āryabhaṭa, *Gola*, 26, Brahmagupta, II, 57, *Sūryasiddhānta*, II, 61, etc.

‡ Manitius, *ibid*, p. 84.

Note.—The perpendicular distance between AA′ and EW is called the 'sine' of the amplitude or the 'Agrā' which is thus calculated:—

$$KL : LS = QO : OM$$

$$\therefore \quad {}^{*}\, R \sin \text{amplitude} = \text{'Agrā'} = KL$$

$$= \frac{LS \times QO}{OM}$$

$$= \frac{R \sin \delta \times R}{R \cos \phi}.$$

It is now evident that the Hindu method is different from the Greek method in this case also. As the triangle KLS is difficult to show in the diagram, it is shown in its projection on the meridian plane in Burgess's translation of the "*Sūrya-siddhānta*," (page 232) and also in Wilkinson and Bāpudev Śastrī's translation of the '*Siddhānta Śiromaṇi*,' p. 175. This has led Braunmühl to assume that the Hindu method of arriving at the equation of ascensional difference and some other equations of spherical Astronomy has its origin in the Analemma of Ptolemy. *A careful study, however, does not justify the identification of Hindu methods with the graphic method of the Analemma, which is deduced from the projections of the position of a heavenly body on the meridian prime vertical and the horizon.* It is being presently shown that what was done out of difficulty in drawing the figures properly has been taken by Braunmühl as a Greek connection.

†*Problem III*.—To find the "time-altitude" equation by the Hindu method.

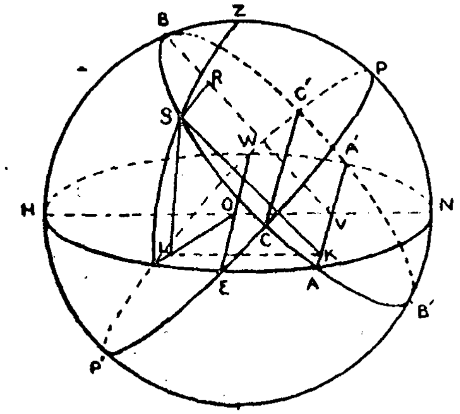
If from any point S on the diurnal circle a perpendicular be drawn to the *Udayāsta-Sūtra* spoken of before, this perpendicular is called the *cheda* or '*iṣṭahṛti*.' The perpendicular from S on the horizon is called '*Śaṅku*,'‡ the sine of the altitude. The line joining the foot of the '*Śaṅku*' and that of the perpendicular on the '*Udayāsta-Sūtra*' goes by the name of '*Śaṅkutala*' and this *Śaṅkutala* lies to the south of the '*Udayāsta-Sūtra*' during the day.

* Āryabhaṭa, *Gola*, 30, etc.

† Āryabhaṭa could not arrive at the true equation. *Cf. Gola*, 28. The correct rules occur in *Pañcasiddhāntikā*, IV, 42-44; *Brāhmasphuṭasiddhānta*, III, 38-36, 26-40; *Sūryasiddhānta*, III, 34-35.

‡ Bhāskara says—यद्ग्रहस्थानादधः शङ्कुः। तस्य तलमुदयास्तसूत्राद्दक्षिणतो भवति ॥ *Gola*, VIII, 39-41. Āryabhaṭa uses the term शङ्कुयम्, *Gola*, 29.

In this figure if AA′ be the *'Udayāsta-Sūtra'* or the intersection of the diurnal circle and the horizon, and S a point on the diurnal circle denoting a position of the sun, SK, SL perpendiculars on AA′ and the horizon respectively; SL is called the *'Śaṅku,'* SK the *'cheda'* and LK the *'Śaṅkutala.'* In this triangle KSL, the angle KSL was recognised to be the latitude of the station.



Thus the triangle SKL is not taken in its projection on the meridian plane. The side SK is taken as formed of two parts. Let CC′ be the line of intersection of the diurnal circle and the 'Six o'clock' circle EPW. Let SK cut CC′ in M. Then,

$$SK = SM + MK$$

Here SM, the 'sine' in the diurnal circle of the complement of the hour angle is given a distinct name *'Kalā'** and MK as explained before is known by the name *'Kujya.'* This *'Kalā'* is constructed from the point S in the diurnal circle. Thus the triangles like SKL were not taken in their projections on the meridian plane as Braunmühl would suggest.

From the triangle KSK, we get,

'Cheda' : *'Śaṅku'* = R : R cos ϕ, where ϕ is the latitude of the observer;

'Śaṅku' is here = R cos Z, Z being the Sun's zenith distance.

$$\therefore \text{ 'cheda'} = \frac{R\cos Z\times R}{R\cos\phi}$$

Now *'Cheda'* = radius of the diurnal circle + *'Kujyā'* − versed sine of the hour-angle in the diurnal circle †O′B + O′V − BR,

$$= R\cos\delta + \frac{R\sin\delta\times R\sin\phi}{R\cos\phi} - \frac{R\text{ vers }H\times R\cos\delta}{R},$$

* Bhāskara's *Grahagaṇita*, VIII, 55.

† O′ is the middle point of CC′ or it is the centre of the diurnal circle ABB′.

$$\left(\text{As in the previous problem, } Kujy\bar{a} = SK = \frac{R\sin\delta\times R\sin\phi}{R\cos\phi}\right)$$

$$\text{or, } \frac{R\cos Z\times R}{R\cos\phi} = \frac{R\cos\delta}{R}\left\{R + \frac{R\sin\delta\times R\sin\phi}{R\cos\phi}\times\frac{R}{R\cos\delta} - R\text{ vers }H\right\}$$

The above equation simplified becomes

$$\cos Z = \sin\delta\sin\phi + \cos\delta\cos\phi\cos H.$$

In this connection we consider the altazimuth equation by the Hindu method.

**Problem IV.*—The Altazimuth Equation:—

Let α denote the azimuth of the sun from the south. In the same triangle SKL in the same figure, we have,

$$LK : SL = R\sin\phi : R\cos\phi,$$

$$\text{or, 'Śaṅkutala' : 'Śaṅku'} = R\sin\phi : R\cos\phi$$

$$\therefore \text{ 'Śaṅkutala'} = \frac{R\cos Z\times R\sin\phi}{R\cos\phi}.$$

Now 'Śaṅkutala' is made up of two parts, namely *'Bāhu'* and *'Agrā,'* of which the former is the distance of L from the observer's East-West line; the *'Agrā'* has been already found.

$$\text{Here 'Bāhu'} = \frac{R\sin Z\times R\cos\alpha}{R}, \text{ and}$$

$$\text{'Agrā'} = \frac{R\sin\delta\times R}{R\cos\phi}.$$

$$\therefore \text{ 'Śaṅkutala'} = \text{'Bāhu'} + \text{'Agrā'}$$

$$\text{or, } \frac{R\cos Z\times R\sin\phi}{R\cos\phi} = \frac{R\sin Z\times R\cos\alpha}{R} + \frac{R\sin\delta\times R}{R\cos\phi},$$

$$\text{or, } R\sin\delta = \frac{R\cos\phi}{R}\left(\frac{R\cos Z\times R\sin\phi}{R\cos\phi} - \frac{R\sin Z\times R\cos\alpha}{R}\right),$$

which is easily seen to be equivalent to

$$\sin\delta = \cos Z\sin\phi - \sin Z\cos\phi\cos\alpha.$$

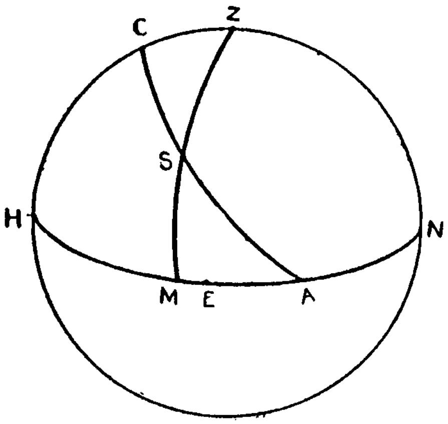
* The equivalent of this, in a particular case, is first found in *Brāhmasphuṭa-siddhānta*, Ch. III, 54-56. *Cf. Sūryasiddhānta*, III, 28-31, also Bhāskara, *Grahagaṇita*, IX, 50-52.

Greek Method:

Ptolemy * has also a method of finding the Sun's altitude at any hour of the day. His method is as follows :—

(*i*) He would find by means of his tables for the times of risings of the signs of the zodiac, the orient ecliptic point. (*ii*) He would then find the culminating point of the ecliptic. (*iii*) He would finally apply Menelaus's theorem in spherics thus :—

Let ASC be any position of the ecliptic, NZC the meridian, NAMH the horizon, Z the zenith and S, the Sun. Here the celestial longitudes of C, S and A are taken to be known; hence ZC and CH are also known.



Now take ZCS for the triangle and HMA to be the transversal; we then have by Menelaus's theorem,

$$\frac{\sin ZH}{\sin HC} \times \frac{\sin CA}{\sin AS} \times \frac{\sin SM}{\sin MZ} = 1$$

or, $$\sin SM = \frac{\cos CZ \times \sin AS}{\sin CA}.$$

It is thus clear that Ptolemy had no direct method for connecting the Sun's altitude and the hour-angle. This method is workable for the problem "given time find the altitude" but is not workable in the converse problem; besides, the calculation of the longitudes of A and C is very cumbrous.

Again, when EA has been found out, taking ZHM for the triangle and CSA for the transversal, we get,

$$\frac{\sin HA}{\sin AM} \times \frac{\sin MS}{\sin SZ} \times \frac{\sin ZC}{\sin CH} = 1,$$ whence AM and thence HM, the azimuth can be found. The method is here also cumbrous, there being no direct connection between altitude and azimuth ; besides the time-element is not avoided.

* Manitius, *ibid*, pp. 118-19.

4. *The Analemma of Ptolemy and the Hindu Method.*

When the Sun's declination is zero and his hour-angle is H, Zeuthen * following the method of the 'Analemma' of Ptolemy, as explained by Braünmuhl, † has deduced the following equations :

(1) $\cos Z = \cos H \cos \phi$

(2) $\tan \alpha = \dfrac{\tan H}{\sin \phi}$.

To these two Heath following Braunmühl, adds

(3) $\tan ZQ = \dfrac{\tan H}{\cos \phi}$ ‡ ,

where Z is the zenith and Q is the point of intersection of the prime vertical and its secondary passing through the Sun and the north-south points.

Zeuthen § points out that later in the same treatise Ptolemy finds the arc 2β described above the horizon by a star of given declination δ', by a procedure equivalent to the formula,

(4) $\cos \beta = \tan \delta' \tan \phi$.

With regard to the 'Analemma' of Ptolemy, it may be noted, as Heath ‖ says, that "the procedure amounts to a method of graphically constructing the arcs required as parts of an auxiliary circle in one plane." Many things may be, in practice, done graphically far more easily than by the theoretical method. Besides, no theoretical calculations occur in the 'Analemma'. Zeuthen, ‖ following the method of this work, has deduced in the general case, the two equations

(5) ‡ $\cos Z = (\cos \delta \cos H + \sin \delta \tan \phi) \cos \phi$.

(6) $\tan \alpha = \dfrac{\cos \delta \sin H}{\dfrac{\sin \delta}{\cos \phi} + (\cos \delta \cos H + \sin \delta \tan \phi) \sin \phi}$.

* Heath, Greek Mathematics, Vol. II, pp. 290-91.
Zeuthen, Bibliotheca Mathematica, 1_3, 1900, pp. 23-27.

† Braunmühl, *ibid*, pp. 12-13.

‡ The Hindu form of this equation is $R \sin ZQ = \dfrac{R \sin H \times R}{\dfrac{\sqrt{R^2 - R^2 \cos^2 H \times R^2 \sin^2 \phi}}{R}}$

Bhāskara's, *Golādhyāya*, Com. on VIII,.67.

§ & ‖ Bjornbo, *loc. cit.*, p. 86.

These equations are suggested to a modern reader from a study of the figures in the 'Analemma.' But neither in this work nor in the 'Syntaxis' are they to be found. With regard to the first four formulæ it is possible that they were recognised by Ptolemy. With regard to the last two, Zeuthen * remarks "mais le texte nen contient rien;" and they were certainly not recognised by Ptolemy.

Besides the tangent function is wholly absent in Greek trigonometry. They are also different in form from those arrived at by the Hindu method as explained before. Thus, it is clear that the Hindu methods are in no way connected with the method of the 'Analemma.'

Even taking for granted that the Hindus followed a method of projection much allied to the method of the 'Analemma,' there is no adequate reason for assuming that their method is derived from any Greek source. Analogy and precedence do not necessarily constitute originality—there is still the chance of a remoter origin from which both the systems drew their inspiration. *The method of the 'Analemma,' as has been already stated, presents a graphical method for constructing the sun's altitude and azimuth from the hour-angle when the Sun's declination is zero—but such a graphical method is nowhere to be met with in the Hindu Astronomy; besides it is generally complex as compared with the elegant Hindu method.* An astronomer who constructs and uses an armillary sphere to arrive at his equations in spherical astronomy and who has not a well-developed spherical astronomy at his command must have to draw perpendiculars from the positions of the heavenly body, not only on the meridian plane, the horizon or on the prime vertical, as the occasion arises, but also on the line of intersection of the diurnal circle with the horizon. Hence Braünmuhl's statement† that the Hindu methods of spherical astronomy have their origin in the 'Analemma,' in spite of his admitting that Indians were first to utilise its methods, is rather far-fetched and tends to take away the honour from the great Indian astronomers, who devised the beautiful Hindu methods.‡ The 'Analemma' as it now exists is a Latin translation from an Arabic version of

* Zeuthen, *loc. cit.*, p. 27.

† Braunmühl, *ibid.* Heath, *ibid.*

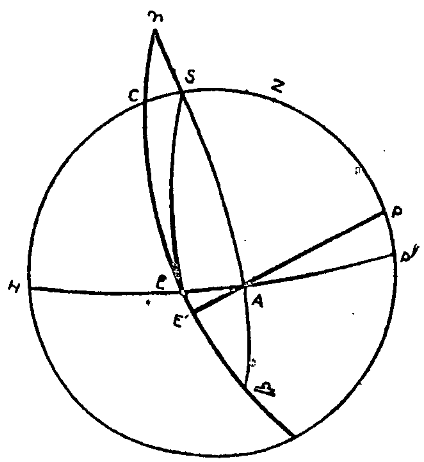
‡ Heath, *ibid*, p. 287.

the original Greek.* We may reasonably doubt that the Arabic version was greatly influenced by the Hindu system.

We now pass on to the consideration of other allied or similar problems in the two systems of astronomy.

Problem V.—To find the angle between the ecliptic and the meridian.

†*Hindu Method*:



Let γSA be the ecliptic, γCE the equator, E the east-point of the orizon. Cut off SH=90° and draw the great circle HEAP′ cutting the meridian P′SCH at the points P′ and H. The aim is to find AP′, but it is enough to find EA since AP′ is the complement of EA′.

Both Āryabhaṭa and Brahmagupta were unable to find EA correctly. Let P be the celestial pole and let PAE′ be the secondary to the equator cutting it at E′. Both the above astronomers were content with the idea that AE=AE′, or that AE=the declination of the point A of the ecliptic which is 90° ahead of S in the above figure. This idea continued till the time of Bhāskara II (1150 A.D.) who found out the correct equation.

He recognised that CS, the declination of S=PP′; P′EH is then the horizon of the station whose north geographical latitude is CS. Also, the 'sine' of EA is the 'Agrā' or the sine of the amplitude of the point A for the latitude CS.

$$\text{R sin EA} = \frac{\text{R sin AE}' \times \text{R}}{\text{R cos CS}}$$

$$= \frac{\text{R sin } (90° + \gamma\text{S}) \times \text{R sin } \omega}{\text{R}} \times \frac{\text{R}}{\text{R cos CS}}$$

* On the influence of the Hindus on Arab mathematics and astronomy; see Alberuni's India, tr. by Dr. E. Sachau, Vol. II, p. 304.

† Āryabhaṭa, *Gola*, 45; *Brāhmasphūṭasiddhānta*, IV, 17; *Sūryasiddhānta*, IV, 25; Bhāskara's *Golādhyāya*, VIII, 21-74, first example in his own commentary.

Or $$\text{R} \sin \text{EA} = \frac{\text{R} \sin (90^\circ + l) \times \text{R} \sin \omega}{\text{R} \cos \delta}$$

where l stands for γS and δ for CS.

Greek Method:

We give below the Ptolemy's method in a slightly modified form.*

Let SHA be the triangle and γCE be the transversal; then we have,

$$\frac{\sin \text{SC}}{\sin \text{CH}} \times \frac{\sin \text{HE}}{\sin \text{EA}} \times \frac{\sin \text{A}\gamma}{\sin \gamma\text{S}} = 1$$

Or $$\frac{\sin \delta}{\cos \delta} \times \frac{\sin 90^\circ}{\sin \text{EA}} \times \frac{\sin (90^\circ + l)}{\sin l} = 1$$

$$\therefore \quad \sin \text{EA} = \frac{\sin \delta \times \sin (90^\circ + l)}{\cos \delta \times \sin l},$$

which is readily transformed into Bhāskara's equation.

The originality of Bhāskara would be readily admitted.

Problem VI.—To find the angle between the ecliptic and the horizon.

Hindu Method:

(A) Āryabhaṭa's method.—It consists of the following steps :—†

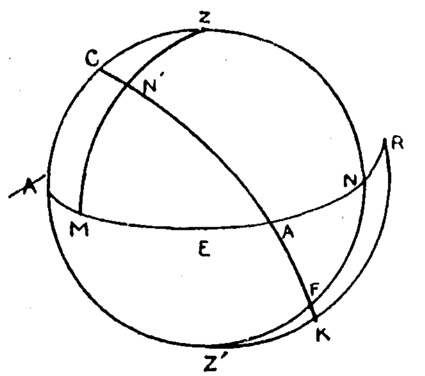
(1) Determination of the orient point of ecliptic.

(2) Finding the sine of its amplitude.

(3) Determination of the culminating point of the ecliptic from the hour-angle of the Sun.

(4) Finding the declination of the culminating point of the ecliptic.

Having obtained the above elements his rule can be followed thus:



In this figure NPZH is the meridian, HMEAN the horizon, CN′A the ecliptic. If N′ be the nonagesimal or the highest point of the ecliptic, the altitude of N′ is the inclination of the ecliptic to the horizon.

Let ZN′M be the vertical through N′, meeting the horizon as M.

When the time is given, the longitudes of A and C can be found out, from which CZ the zenith distance of C and EA the amplitude of the orient ecliptic point can be determined.

Here HM = EA.

According to Āryabhaṭa,

$$\text{R} \sin \text{CN}' = \frac{\text{R} \sin \text{CZ} \times \text{R} \sin \text{HM}}{\text{R}}$$

and $\text{R} \sin \text{ZN}' = \sqrt{(\text{R} \sin \text{CZ})^2 - (\text{R} \sin \text{CN}')^2}$.

This is only an approximate rule. As expressed here,

$$\text{R} \sin \text{ZN}' = \frac{\text{R} \sin \text{CZ} \times \text{R} \cos \text{HM}}{\text{R}} \text{ approximately}$$

$$= \frac{{}^*\text{R} \sin \text{CZ} \times \text{R} \cos \text{HM} \times \text{R}}{\text{R} \times \text{R} \cos \text{CN}'} \text{ accurately}$$

$$= \frac{\text{R} \sin \text{CZ} \times \text{R} \cos \text{HM}}{\text{R} \cos \text{CN}'}.$$

(B) The method of Brahmagupta:†

Brahmagupta would also first determine the orient ecliptic point A. Then he subtracts 90° from the longitude of A. Thus having the longiitude of N′, he next finds the part of the day elapsed of N′; from which by the time-altitude equation discussed above, he finds ZN′. This is of course more accurate than that of Āryabhaṭa. Bhāskara ‡ here follows Brahmagupta.

Greek Method:

Let the ecliptic CN′A cut the lower half of the meridian at F. Ptolemy takes AK along the ecliptic = 90° and AR along the horizon = 90°; then the great circle passing through R and K passes through the nadir Z′. Now take Z′FK for the triangle and ANR for the transversal, then by Menelaus' theorem.§

$$\frac{\sin \text{FN}}{\sin \text{NZ}'} \times \frac{\sin \text{Z}'\text{R}}{\sin \text{RK}} \times \frac{\sin \text{KA}}{\sin \text{AF}} = 1$$

$$\therefore \quad \sin \text{RK} = \frac{\sin \text{FN}}{\sin \text{AF}} = \frac{\cos \text{FZ}'}{\sin \text{AC}} = \frac{\cos \text{CZ}}{\sin \text{AC}} = \frac{\sin \text{CH}}{\sin \text{AC}}$$

Or $$\sin \text{MN}' = \frac{\sin \text{CH}}{\sin \text{AC}}.$$

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* Manitius, *ibid*, Book I, pp. 104-06.

† Āryabhaṭa, *Golā*, 33; *Sūryasiddhānta*, V. 5-6.

* This correction was perhaps first noticed by Raṅganātha (1603 A.D.) in his commentary on the *Sūryasiddhānta*.

† *Brāhmasphuṭasiddhānta*, V, 3.

‡ *Grahagaṇita*, XII, 3-4.

§ Manitius, *ibid*, pp. 110-11.

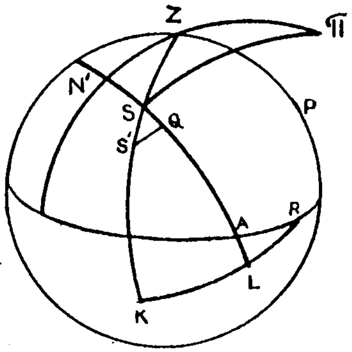
Here Ptolemy's equation is simpler than that of Āryabhaṭa; hence they must be independent of each other.

Problem VII.—To find the angle made by the vertical through any point of the ecliptic with the latter.

This problem is considered by Ptolemy but it is not considered separately in Hindu Astronomy, but from the rule for parallax in longitude, the rule for its calculation can be deduced.

Hindu Rule:

In the adjoining figure S represents the true position of Sun and S′ the Sun's position as depressed by parallax. N′SA is the ecliptic. If from S′, S′Q be drawn perpendicular to the ecliptic, then, if P is the horizontal parallax,



$$SQ = SS' \times \frac{R \cos S'SQ}{R}$$

$$= \frac{P \times R \sin ZS}{R} \times \frac{R \cos S'SQ}{R}$$

$$= \frac{{}^{*}P}{R} \sqrt{(R \sin ZS)^2 - R(\sin ZN')^2}$$

$$= \frac{{}^{\dagger}P}{R^2} \times R \sin N'S \times R \cos ZN', \text{ where } N' \text{ is the nonagesimal.}$$

Thus R cos S′SQ is seen to be

$$= \frac{R \sin N'S \times R \cos ZN'}{R \sin ZS}.$$

The Hindu method is fully described by Bhāskara in his '*Golādhyāya*,' VIII, 12-25. The truth of the Hindu rule for R cos S′SQ is easily seen from the spherical triangle π ZS, where π is the pole of the ecliptic.

* Āryabhaṭa, *Gola*, 34; *Pañcasiddhāntikā*, IX, 22; *Brāhmasphūṭasiddhānta*, XI, 23.

† *Brāhmasphūṭasiddhānta*, V, 4-5; *Sūryasiddhānta*, V, 7-8; Bhāskara, *Grahagaṇita*, XII, 4.

Greek Method:

* Ptolemy takes SK and SL = 90° each, along the vertical circle ZSEK and the ecliptic N′SA. The great circle through K and L cuts the horizon at R which is the pole of the vertical circle. He takes SKL for the triangle and EAR for the transversal, then

$$\frac{\sin SE}{\sin EK} \times \frac{\sin KR}{\sin LR} \times \frac{\sin LA}{\sin AS} = 1$$

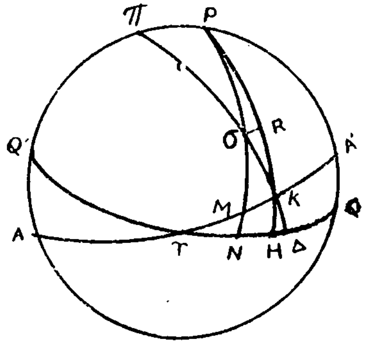
or $$\sin LR = \frac{\cos ZS \times \cos AS}{\sin ZS \times \sin AS}$$

or $$\cos S'SQ = \cot ZS \times \cot AS = \tan SE \cot AS.$$

The Hindu and the Greek rules are altogether different both in form and method. There can, therefore, be no question of any connection between them.

Problem VIII.—To convert the celestial longitude of a heavenly body into its polar longitude.

If σ be the position of a star, γK and σK are the celestial longitude and the celestial latitude respectively; γM and σM are the polar longitude and polar latitude; γN and σV are the right ascension and declination of the star.



Hindu Method:

All Indian astronomers attempt at finding MK which, subtracted from, or added to, γK the celestial longitude, gives γM the polar longitude.

According to Āryabhaṭa,†

$$MK = \frac{\sigma K \times R \text{ vers } \gamma K \times R \sin \omega}{R^2}.$$

Brahmagupta ‡ makes a distinct improvement on Āryabhaṭa and gives his rule for finding the projection MK on the celestial equator.

* Manitius, *ibid*, p. 119.

† Āryabhaṭa, *Gola*, 36.

‡ *Brāhmasphuṭasiddhānta*, X, 17.

If P be the celestial pole, PKH the secondary to the equator, Brahmagupta says that,

$$NH = \frac{\sigma K \times R \sin(\gamma K + 90^\circ) \times R \sin \omega}{R}.$$

If from σ, σR is drawn perpendicular to PKH, it is evident that,

$$R \sin \sigma R = \frac{R \sin \sigma K \times R \sin \sigma KR}{R}.$$

According to Āryabhaṭa and Brahmagupta, as explained before,

$$R \sin \sigma KR = \frac{R \sin(\gamma K + 90^\circ) \times R \sin \omega}{R}.$$

Hence Brahmagupta intends that,

$$NH = \sigma R = \frac{\sigma K \times R \sin \sigma KR}{R}$$

which is rather a big assumption. He then directs the finding of the part of the ecliptic of which σR or NH is the projection on the equator thus approximately to MK.

Āryabhaṭa, Brahmagupta * and the modern *Sūryasiddhānta* take the declination $\sigma N = \sigma K + KH$ where σK is small. They do not consider the case where σK is large.

Bhāskara alone gives us fairly correct rules for this transformation of co-ordinates.

In order to find σN, he would multiply

σK by $\frac{R \cos \sigma KP}{R}$; according to him,

$$\sigma N = \frac{\sigma K \times R \cos \sigma KP}{R} + KH. \dagger$$

This is a decided improvement on Brahmagupta's corresponding rule. The declination σN obtained would be very nearly accurate.

Having obtained σN, Bhāskara † then directs the finding of NH, thus,

$$NH = \frac{\sigma K \times R \sin \sigma KP}{R \cos \sigma N}.$$

* *Brāhmasphūṭasiddhānta*, X, 15 ; *Sūryasiddhānta*, II, 58.

† Bhāskara, *Grahagaṇita*, XIII, 3.

He then directs the finding of MK on the ecliptic of which NH is the projection by means of the times of rising of the signs of the zodiac on the equator.

Thus, the Hindu methods show a beginning and development only. The Greek method as given by Ptolemy is mathematically accurate.

**Greek Method* :

To transform the celestial longitude and celestial latitude to right ascension and declination.

Let the great circle $\pi\sigma K$ meet the equator at $\triangle$. Ptolemy would then form the given value of γK find $\gamma\triangle$ and $\triangle K$ by using his tables for the rising of signs of the zodiac on the equator. He then takes $\pi P \sigma$ for the triangle and $\gamma N \triangle Q$ for the transversal. The Menelaus' Equation, then, is

$$\frac{\sin \pi Q}{\sin QP} \times \frac{\sin PN}{\sin N\sigma} \times \frac{\sin \sigma\triangle}{\sin \triangle\pi} = 1. \dagger$$

Here $\pi Q = 90^\circ + \omega$, $QP = 90^\circ$, $PN = 90^\circ$, $\sigma\triangle = \sigma K + K\triangle$. $\pi\triangle = 90^\circ + K\triangle$, whence $N\sigma$ is obtained.

He next takes PNQ for the triangle and $\pi\sigma\triangle$ for the transversal,

$$\therefore \quad \frac{\sin P\pi}{\sin \pi Q} \times \frac{\sin Q\triangle}{\sin \triangle N} \times \frac{\sin N\sigma}{\sin \sigma P} = 1.$$

Here $P\pi = \omega$, $\pi Q = 90^\circ + \omega$, $Q\triangle = 90^\circ - \gamma\triangle$.

Hence the above equation gives him $\triangle N$. Now, $\gamma N = \gamma\triangle - \triangle N$.

It is almost needless to say that neither in the method nor in the rules is there any agreement between the Hindu and Greek spherical astronomy in the solution of this problem.

Mr. Kaye's view : ‡

As to Mr. Kaye, it appears that he has not been able to find a method in the translation of the *Sūryasiddhānta* by Burgess. The figures of his paper referred to before do not show the "*Akṣa-kṣetras*" even in their projections on the meridian place. He refers to Braunmühl's History of Trigonometry but does not appear to have been able to follow him in his "Methode der indischen Trigonometrie." Kaye however is not slow in belittling Hindu trigonometry when he says :—"The Indian astronomers employed

* *Ibid*, XIII, 4.

† Manitius, *ibid*, Vol. II. Achtes Buch, pp. 84-85.

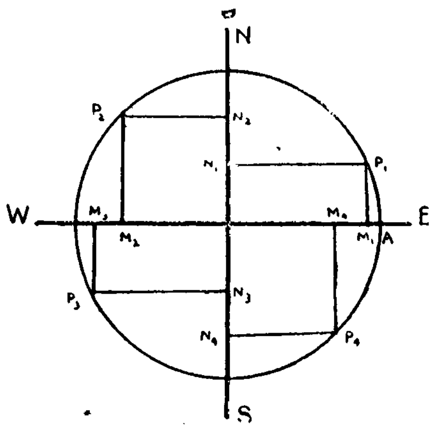
‡ J.A.S.B., N.S., XV, p. 154.

the sine function principally and the versed sine occasionally; they never employed the tangent function; and generally, but not always, preferred to employ the sine of the complementary angle rather than the cosine functions."

It is evident that Kaye never understood the meaning of the Indian functions of 'sine' and 'cosine.' These functions are fully explained by Bhāskara * when he says:—

"तस्य बिन्दोः प्राच्यपरायाश्च यदन्तरं सादोर्ज्या ।
बिन्दोर्याम्योत्तरायाश्च यदन्तरं साकोटिज्या ॥"

In the adjoining figure, of the arc AP_1, P_1M_1 is the "sine" and P_1N_1 is the "cosine;" of AP_2, P_2M_2 is the "sine" and P_2N_2 is the "cosine;" of AP_3, P_3M_3 is the "sine" and P_3N_3 is the "cosine;" etc. It is evident that a better definition of these functions was never given.



Kaye's motive is clear: his chief aim is to show the Greek connection in everything Indian. No doubt, he has shown great ingenuity in this direction. Instances might be multiplied but the scope of this paper would not permit this.

5. *Conclusion.*

We have now come to the end of the present paper. We have seen that some of the solutions of Āryabhaṭa are imperfect, of Brahmagupta the solutions are more accurate, while those of Bhāskara are generally mathematically correct. The date of the scientific Hindu Astronomy is indeed 499 A.D., while that of the Syntaxis is about 150 A.D. It is by these shortcomings and differences in the methods, new ideas (*e.g.*, the idea of the differential calculus) * and the like, that we can safely say that Hindu Astronomy in its scientific form, although of a later date than the "Syntaxis" of Ptolemy, is original and not borrowed from foreign source.† There is evidence that some crude form of Greek astronomy was transmitted to India and went by the name of the "Romaka" or the "Pauliśa" Siddhānta, prior to the time of Āryabhaṭa but our great Indian astronomers, Āryabhaṭa with his pupils, Varāha-Mihira and Brahmagupta, had to construct a new science altogether.

* Bhāskara, *Grahagaṇita*, II, 88-21, Commentary. The passage may be translated thus: "Of that point the distance from the east-west line is the 'sine' and the distance of the point from the north-south line is the "cosine."

* P. C. Sengupta, History of the Infinitesimal Calculus in Ancient and Medieval India in the "Jahresbericht D. Math-Vereinigung" published in Feb., 1931.

† The entire set of Hindu astronomical constants is also different from that in the Greek system. *Vide* the present writer's paper, "Āryabhaṭa" (pp. 39, 43), Journal of the Department of Letters, Vol. XVII, Calcutta University Press.

APPENDIX III

Hindu Epicyclic Theory

An attempt is made here to describe the ideas of planetary motion which the Hindu astronomers had as to the motion of "planets" or wandering bodies among the stars. The earliest system of Hindu astronomy of which we have a definite account is the *Jyotiṣa Vedāñga*, dating from about 1400 B.C. In this system there is nothing of a clear conception of planetary motion. It was a simple luni-solar system of calendar making for fixing the times for the performance of Vedic sacrifices. The real scientific Hindu astronomy dates from the time of Āryabhaṭa I (born 476 A.D.), who must now be regarded as its sole originator.* The foreign systems of astronomy that had come to India before his time, were undoubtedly the *Romaka* and the *Pauliśa Siddhāntas* of the *Pañca-siddhāntikā.*

The *Romaka Siddhānta* was no doubt the transmitted Greek astronomy and the *Pauliśa* and perhaps also the *Vasiṣṭha Siddhānta* as summarised in the *Pañcasiddhāntikā* were probably of Babylonian origin. Lṭāadeva, the expounder of the *Romaka* and the *Pauliśa Siddhāntas*, it has now been established, was one of the first batch of pupils of Āryabhaṭa I. According to Alberuni, the *Sūryasiddhānta* which is admittedly of *Asura* (or Babylonian) origin, was composed by Lāṭadeva.† The *Sūryasiddhānta* as given in Varāha's work was probably a recast of an older work by Varāha himself, based on the teachings of Āryabhaṭa. Whether Alberuni be right or wrong, Lāṭadeva got the appellation of सर्वसिद्धान्तगुरु, *i.e., teacher of all the systems of siddhāntas.* It is now definitely known that Āryabhaṭa I was the teacher of the famous array of Indian astronomers, *viz.*, Niśśaṅku, Lāṭadeva, Pāndurangasvamī, Bhāskara I and of others of lesser fame.

* P. C. Sengupta, Āryabhaṭa the Father of Indian Epicyclic Astronomy, *Calcutta University Journal of the Department of Letters*, Vol. XVIII.

† Alberuni's India, Vol. I, Ch. XIV, p.1 Translated by Sachau.

The transmitted Greek astronomy in the shape of the *Romaka Siddhānta* had no trace of the epicyclic construction for planetary motion as we find in Ptolemy's Syntaxis.* A comparison of the astronomical constants of the Greek and the Hindu systems, points unmistakably to the conclusion that the Hindu constants as determined by Āryabhaṭa I and his successors, are almost in all cases different from those of the Greeks. In respect to the elements therefore the originality of the Hindu astronomers will be admitted on all hands. As regards the originality of doctrine, materials available at present makes it impossible for us to ascertain what part of the doctrine also belongs to the Hindu astronomers. In the present paper we shall rely mainly on Āryabhaṭa I.

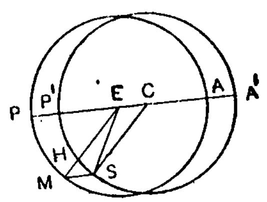
Apparent Motions of the Sun and Moon.—Āryabhaṭa says, "All planets move in eccentrics to their orbits at the mean rates of angular motion, in the direction of the signs of the zodiac from their apogees (or aphelia) and in the opposite directions from their *śighroccas.*"

"The eccentric circles of planets are equal to their concentrics, and the centre of the eccentric is removed from the centre of the earth."

"The distance between the centre of the earth and the centre of the eccentric is equal to the radius of the planet's epicycle; on the circumference (whether of the epicycle or of the eccentric) the planet undoubtedly moves with the mean motion."†

Here the central idea was that there was no doubt that the planets moved uniformly in circles round the earth, if the motion appeared to be variable, it was due to the fact that the centres of such circle (*i.e.*, the eccentric circles) did not coincide with the centre of the earth.

Let E represent the centre of the earth, APM the sun's circular orbit or concentric; let A and P be the apogee and the perigee respectively. From EA, cut off EC equal to the radius of the sun's epicycle. With centre C and radius equal to EA describe the eccentric A′P′S cutting AP and AP produced at P′ and A′. Here A′ and P′ are the real apogee and perigee of the sun's orbit. Let PM and P′S be any two equal arcs measured from P and P′.



* P. C. Sengupta, Āryabhaṭa, already referred to.

† *Kālakriyā*, 17-19. Cf. *Brāhmasphuṭa-siddhānta*, XIV, 10-12; Bhāskara II, *Golādhyāya*, V, 7, 10-32.

The idea is that the mean planet M and the apparent sun S move simultaneously from P and P′ in the counterclockwise direction along the concentric and the eccentric circles respectively. They move with the same angular motion and arrive simultaneously at M and S.

Here EM and CS are parallel and equal, hence MS is also equal and parallel to EC. Let SH be drawn perpendicular to EM.

The angle PEM is the mean anomaly and the angle P′ES the true anomaly; the angle SEM is the equation of the centre, is readily seen to be + from P′ to A′ and − from A′ to P′. Thus as regards the character of the equation, the eccentric circle is quite right. We now turn to examine how far it is true as to the amount.

Let the angle SEM be denoted by E and the angle ∠PEM = ∠P′CS = θ; EP = CP′ = a; EC = MS = p, then

$$\tan E = \frac{SH}{HE} = \frac{p \sin\theta}{a - p\cos\theta}$$

$$\therefore \quad E = \frac{p}{a}\sin\theta + \frac{p^2}{2a^2}\sin 2\theta + \frac{p^3}{3a^3}\sin 3\theta + \ldots \ldots \ldots$$

Now the true value of E in elliptic motion is given by

$$E = \left(2e - \frac{e^3}{4}\right)\sin\theta + \tfrac{5}{4}e^2\sin 2\theta + \frac{13e^3}{12}\sin 3\theta \,* \ldots \ldots \ldots$$

If we now put $\frac{p}{a} = 2e - \frac{e^3}{4}$, as a first approximation $\frac{p}{a} = 2e$.

Hence $\frac{p^2}{2a^2} = 2e^2$, which is greater than $\tfrac{5}{4}e^2$ by $\tfrac{3}{4}e^2$. In the case of the sun if the value of p be correctly taken, the error in the co-efficient of the second term becomes +8′; similarly in the case of the moon the corresponding error becomes +8′.

Again if $\frac{p}{a} = 2e$, what is the centre of the eccentric circle is the empty focus of the ellipse, or that the ancient astronomers practically took the planets to be moving with uniform angular motion round the empty focus. This was not a bad approximation.

* Godfray's Astronomy, p. 149.

Also ES = r = EH approximately

$$\therefore \quad r = a\left(1 - \frac{p}{a}\cos\theta\right)$$

but in elliptic motion,

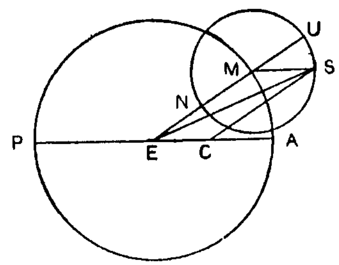
$$r = a(1 - e\cos\theta).*$$

Hence the error is not very considerable here also.

This is the way in which the ancient astronomers, both Greek and Hindu, sought to explain the inequalities in the motion of the sun and the moon. In the case of the moon, these astronomers took the co-efficient $2e - \frac{e^3}{4} = 300'$ nearly; the modern value of it is 377′ nearly. The reason for this has been pointed out already in appendix I, that the moon was observed correctly only at times of eclipses. At the eclipses or syzygies the evection term of the moon's equation diminishes (numerically) the principal elliptic term by about 76′.

We have thus far explained the idea of planetary motion of the ancients under the eccentric circle construction. The same however is also explained under the epicyclic construction.

Let AMP be the circular orbit of the sun, having E the centre of the earth for the centre. Let the diameter AEP be the apse line, A the apogee and P the perigee. Let M be the mean position of the sun in the orbit. With M as the centre describe the epicycle UNS. Let EM cut the epicycle at N and U. Now the construction for finding S the apparent sun is thus given:—



Make ∠UMS = ∠MEA, the arc US is measured clockwise whereas the arc A to M is measured counterclockwise.

From this construction MS is parallel to EA. If EC be measured = MS the radius of the epicycle, along EA towards the apogee, then CS is a constant length and C is a fixed point. Hence the locus of S is an equal circle with the centre at C. Thus both the eccentric, and the epicycle and the concentric combined, led to the same position and orbit for S.†

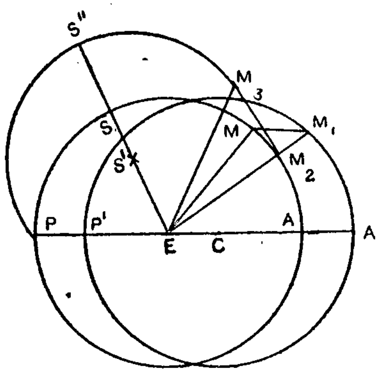
* Godfray's Astronomy, p. 149.

† Bhāskara II, *Gola*, V, 30-32.

It was thus usual to explain the planetary motion under both the assumed constructions; and both gave the same position for a planet. The eccentric circle construction appears to be the earlier in the history of astronomy and the latter was later. If the former construction can be traced to Apollonius of Perga who did so much to develop the "conic sections" as a science, the reason why he preferred the eccentric circle to the ellipse, appears to be that either that this planetary construction was always deep-rooted in the minds of men or that he was carried by the idea that "the circle was the most perfect curve." We are inclined to the view that the eccentric circle idea was transmitted from Babylonia to Greece. We now pass on to consider the Hindu construction for the position of superior and inferior planets.

Superior Planets.—With regard to the five planets, Mercury, Venus, Mars, Jupiter and Saturn, the Hindu astronomers give only one construction for finding the apparent geocentric position. Each of these "star planets" is conceived as having twofold planetary inequalities: (*i*) the inequality of apsis, (*ii*) the inequality of the *śīghra*. With regard to the superior planets, the *śīghra* apogee or the *śīghrocca* coincided with the mean position of the sun. As Varāhamihira observes, "of the other planets beginning with Mars the sun is the so-called *śīghra*.*

Let AMSP be the concentric of which the centre E is the same as that of the earth; $A'M_1P'$ the eccentric circle of apsis of a superior planet, of which the centre is C; A, M, S, P be respectively the apogee, the mean planet, the direction of the *śīghra*, and the perigee in the concentric; A', M_1, P' be the apogee, the planet as corrected by the equation of apsis, P' the perigee in the eccentric. The arc $AM = \text{arc } A'M_1$; MM_1 is parallel and equal to EC. As used before both the concentric and the eccentric are of the same radius.



* *Pañcasiddhāntikā*, XVII, 1.

Here the mean planet M in the concentric is taken to be deflected to M_1 due to the true motion in the eccentric circle. Join EM_1 cutting the concentric at M_2. Now let ES be joined and let S' be taken along ES, such that

$$\frac{ES'}{ES} = \frac{\textit{śīghra}\text{ periphery of the planet in degrees}}{360}$$

$$= \frac{\text{Sun's mean distance from the earth *}}{\text{Planet's mean distance from the sun or the earth}};$$

ES' thus determined is called the radius of the *śīghra* epicycle of the superior planet.

With S' as the centre and radius equal to ES or EA, describe another circle which is called the *śīghra* eccentric cutting ES produced at S''. Now measure the arc $S''M_3$ in the eccentric $= SM_2$ in the concentric. The apparent superior planet is seen in the direction EM_3 from the earth. This is the construction used in Hindu astronomy for calculating the geocentric longitude of any star planet.

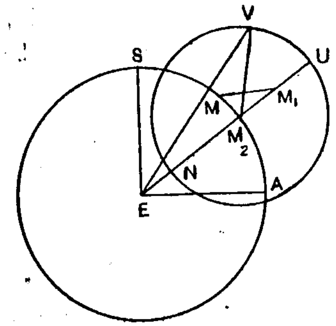
It is evident in the case of a superior planet that the eccentric having S' for the centre and whose radius $= EA = R$ the standard radius for any circular orbit, is the mean orbit of the planet and S' the mean position of the sun. In other words in the case of a superior planet the *śīghra* eccentric represents the mean orbit round the sun. If the parallelogram $CES'C'$ be constructed, then an equal circle described with C' as the centre is the apparent eccentric orbit of the superior planet.

In the actual method of calculating the geocentric longitude of a 'star planet,' there are four operations given, the first two of which have the effect of changing the arc MA or rather the point A.† The last two operations relate to the two displacements MM_1 and M_2M_3. We have here followed solely the construction by the eccentric circles; the same geocentric position of a superior planet could be equally well obtained by the epicyclic construction. In describing the construction for finding the position of an inferior planet we shall follow the epicyclic construction only.

* P. C. Sengupta, Translation of the *Āryabhaṭīya*, *Kālakriyā*, pp. 36-38, Journal of the Department of Letters, Vol. XVI.

† Translation proper, pp. 58-60.

Inferior Planets.—Let E be the centre of the earth, AMS the orbit of a mean inferior planet or the mean sun. EA the direction of the apogee of apsis and ES that of the *śighra*. The inequality of the apsis takes the mean geocentric planet from M to M_1, such that MM_1 is parallel to EA. Let EM_1 be joined cutting the concentric at M_2; M_2 is taken as the centre of the *śighra* epicycle or the real circular orbit in which the apparent planet moves.



With M_2 as the centre and the radius of the inferior planets' *śighra* epicycle as radius, describe the circle NVU which is here the *śighra* epicycle or the real circular orbit. In it draw the radius M_2V parallel to ES; then VP is the geocentric position of the inferior planet.

Here the first displacement MM_1 is due to the inequality of apsis and is for finding the position of M_2 the centre of the real circular orbit. The idea was that the apparent planet moved in a circular orbit of which the centre was very near the mean position of the sun, orbit of which the centre was calculated to determine the first operation in this construction was calculated to determine the centre of this so-called circular orbit of an inferior planet.

The *śighra* of an inferior planet moves round the earth at the same mean rate in which the inferior planet moves round the sun; hence the line ES in this figure is always parallel to the line joining the sun to the mean heliocentric inferior planet, and in our construction it is parallel to M_2V.*

Such in brief is an outline of the Hindu idea of planetary motion as taught by Āryabhaṭa I, Brahmagupta and Bhāskara II and others. In order to avoid complexity we have omitted the details. In our paper Āryabhaṭa we have indicated how the twofold inequalities were separated in case of a superior planet by the ancients.† In the case of inferior planets, the method perhaps was that of finding by observation when and where their eastern and western elongations from the mean position of the sun were equal. These were the real methods of the ancients and that there is no doubt that the Hindu astronomers followed the same methods in finding the elements of the orbits anew.

* P. C. Sengupta, Translation of the *Āryabhaṭīya*, *Kālakriyā*, 17, pp. 35-36.

† P. C. Sengupta, Āryabhaṭa, pp. 45-52.

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120	14	hypotcnuse	hypotenuse
124	13	*kālāṇśa*	*kālāṁśa*
141	2	स्फटखण्डका:	स्फुटखण्डका:
144	16	ib	its